Seat No.: __________ Enrolment No.____________

GUJARAT TECHNOLOGICAL UNIVERSITY

ME Semester –I Examination Feb. - 2012

Subject code: 710703N **Date: 17/02/2012**

Subject Name: Modern Control System

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Attempt All (Each question carries One mark) 07

1. The state variable approach is applicable to
   a. Only linear time invariant system
   b. Linear time invariant and time varying system
   c. Linear as well as non linear system
   d. All systems
2. The number of state variables is equal to
   a. No. of integrators present in the system
   b. No. of differentiators present in the system
   c. Sum of integrators and differentiators present in the system
   d. None of above
3. The eigen values of the state model is the same as
   a. Open loop poles
   b. Closed loop poles
   c. Both open loop and closed loop poles
   d. None of above
4. The $n \times n$ matrix is said to be nonsingular if the rank of the matrix (R) is
   a. $R = n$
   b. $R \neq n$
   c. $R = n/2$
   d. $R = 2n$
5. Define state, and state variable.
6. Mention the condition for selecting the state variable for the system.
7. Write the state and output equation with proper name and dimension of each matrix.

(b) Attempt following.

1. What are the advantages of state space modeling technique over the transfer function modeling technique in control system analysis? 03
2. Write and prove the properties of the State Transition Matrix (STM). 04

Q.2 **(a)** Obtain the co-relation between the state space equation and transfer function. Obtain the transfer function for the system given in state model form as 07

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$[x(0)] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b) Obtain the state model of the system given in Figure 1. Consider the state 07 variables as i_1, i_2 and v_c

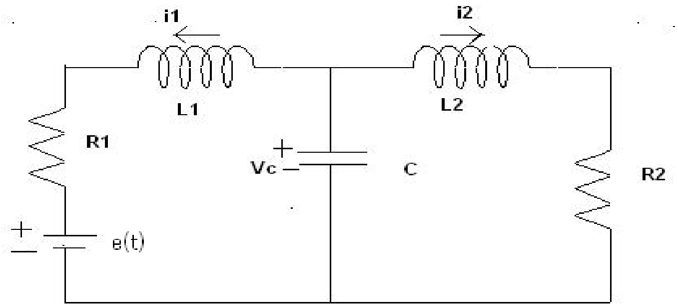

Figure 1

OR

(b) Obtain the state model of the Armature Controlled DC motor. 07

Q.3 **(a)** Explain the Caylay-Hemilton theorem and show how it is useful for solving the STM ? 07

(b) Obtain the time response of the system given by 07

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$[x(0)] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad u(t) = 1, \quad t > 0$$

OR

Q.3 **(a)** Obtain First companion form, Second companion form and Jordan canonical form with state diagrams for a given transfer function 07

$$G(s) = (s+3) / (s^3 + 9s^2 + 24s + 20)$$

(b) Prove that the Eigen values are invariant under a linear transformation. 07

Q.4 **(a)** Derive the condition for checking the Controllability for a given system. 07

(b) Check for the controllability and the Observability of the system given by 07

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

OR

Q.4 **(a)** State the Observability and also derive the condition for checking the same for a given system. 07

(b) Show that the system given below is not observable 07

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q.5 **(a)** Explain the Lyapunov stability criterion for the Linear Time Invariant System. 07

(b) Define the Sylvester's criterion for checking the definiteness and also check the definiteness of the following functions 07

1. $v(x) = x_1^2 + 2x_2^2$

2. $v(x) = 10x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_1x_3$

OR

Q.5 **(a)** Define asymptotic stability and also check for the asymptotic stability of the system given by 07

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b) Design a state feedback controller gain using pole placement technique for the system given by 07

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

The desired pole locations of the closed loop system are $s = -3$ and $s = -5$

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