**Seat No.: _____** **Enrolment No._____**

# GUJARAT TECHNOLOGICAL UNIVERSITY

## ME Semester –I Examination Feb. - 2012

**Subject code: 714101N** **Date: 11/02/2012**

**Subject Name: Mathematical Methods in Signal Processing**

**Time: 10.30 am – 01.00 pm** **Total Marks: 70**

## Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Obtain Z transform of x(n) = u(n) and x(n) = $0.5^n$ u(n). **07**

**(b)** Compute transpose, rank and trace for the matrix *A*. **07**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

**Q.2** **(a)** Show representations for continuous time signal,discrete time signal. Obtain Fourier Transform for x(t) = δ(t) and x(t) = sin($\omega_0$t) /(π t). **07**

**(b)** Show that the matrices are self - adjoint **07**

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 6 & 2 \\ 0 & 2 & -3 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 2 & 2+3j & 5-j \\ 2-3j & 4 & 6+4j \\ 5+j & 6-4j & 9 \end{bmatrix}$$

**OR**

**(b)** State Cayley-Hamilton theorem. Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, then using Cayley Hamilton theorem Calculate $A^2 - 7A + 6I$. **07**

**Q.3** **(a)** Represent Moving Average (MA) signal and Auto Regressive (AR) signal. How can one obtain autocorrelation function for these signals ? **07**

**(b)** Explain about Hidden Markov Model. **07**

**OR**

**Q.3** **(a)** For $x, y \in$ R, Prove the triangle inequality in the form $|x+y| \le |x| + |y|$ , what is the condition for equality? **04**

**(b)** "A linear operator $A : X \to Y$ is bounded if and only if it is continuous". Justify the statement. **04**

**(c)** Consider a data sequence $\{x(t)\}$, the correlation matrix R is **06**

$$R = \begin{bmatrix} .5 & .3 \\ .3 & .5 \end{bmatrix}$$

and the cross-correlation vector p with a desired signal is $p = \begin{bmatrix} .2 \\ .5 \end{bmatrix}$

Determine the optimal weight vector.

**Q.4** **(a)** Explain about maximum likely hood detector and log-likelihood function. **07**

**(b)** Show that the diagonal elements form the eigenvalues of matrix *A*, if *A* is triangular. **04**

**(c)**

Prove Parseval's theorem for Fourier transforms: If $y_1(t) \leftrightarrow Y_1(w)$ *and* $y_2(t) \leftrightarrow Y_2(w)$, **03**

then $\int_{-\infty}^{\infty} y_1(t)\overline{y_2}(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y_1(w)\overline{Y_2}(w)dw.$

**OR**

**Q.4** **(a)** Find eigen values and corresponding eigen vector of the matrix *A* **07**

$$A = \begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$$

**(b)** For matrix T in block triangular form **07**

$$T = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix}$$

show that $\lambda(T) = \lambda(T_{11}) \cup \lambda(T_{22})$

**Q.5** **(a)** Explain about Adaptive Filter with its block diagram. **07**

**(b)** Explain about ARMA parameter estimation method. **07**

**OR**

**Q.5** **(a)** Let $x_t, y_t$ *and* $v_t$ be discrete-time random processes with **07**

$y_t = x_t + v_t$ and $S_v(z) = 1$, $S_x(z) = \frac{b(z)}{a(z)}$

Where b(z) and a(z) are polynomials in z with the degree of b(z) strictly lower than the degree of $a(z)$. Furthermore,

assume $R_{xv}(t) = 0$. Show that $H(z) = 1 - \frac{1}{S_y^+(z)}$ holds

**(b)** Explain about linear predictor with block diagram. **07**

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