Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
ME - SEMESTER I - EXAMINATION – SUMMER 2017

Subject Code: 2710710 **Date:08/05/2017**
Subject Name: Applied Linear Algebra
Time: 02:30 pm to 05:00 pm **Total Marks: 70**
Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Let $T: R^3 \to R^3$ be the linear operator defined by the formula $T(x_1, x_2, x_3) = (3x_1 + x_2, -2x_1 - 4x_2 + 3x_3, 5x_1 + 4x_2 - 2x_3)$ **07**
Determine the whether T is one to one, if so, find $T^{-1}(x_1, x_2, x_3)$

(b) State and prove rank – nullity theorem **07**

Q.2 **(a)** **(1)** Express the vector $v = (6,11,6)$ as a linear combination of $v_1 = (2,1,4)$, $v_2 = (1,-1,3)$ and $v_3 = (3,2,5)$ **07**
(2) Determine whether the following set of vectors form a basis for R^3.
$(1,1,1),\ (1,\ 2,\ 3),\ (2,-1,1)$

(b) Let $T: R^3 \to R^3$ be multiplication by A. Determine whether T has an inverse. If so, find $T^{-1}\left(\left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right]\right)$ where $A = \begin{bmatrix} 1 & 4 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ **07**

OR

(b) Determine whether the following functions are linear transformation : **07**
(1) $T: M_{mn} \to M_{nm}$, where $T(X) = X^T$
(2) $T: M_{22} \to R, T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a^2 + b^2$

Q.3 **(a)** If $T: R^2 \to R^3$ be the linear transformation defined by **07**

$$T\left(\left[\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$$

Find the matrix of the transformation T with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3, where

$$u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(b) Apply the power method to the following symmetric matrix to obtain the dominant eigen value by taking the initial approximation as $x_0 = [1\ 1\ 1]^T$ **07**

$$\begin{bmatrix} 0.49 & 0.02 & 0.22 \\ 0.02 & 0.28 & 0.20 \\ 0.22 & 0.20 & 0.040 \end{bmatrix}$$

OR

Q.3 **(a)** Let $T: R^4 \to R^3$ be the linear transfomration given by the formula $T(x_1, x_2, x_3, x_4)$=$(4x_1 + x_2 - 2x_3 - 3x_4, 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4)$ **07**

(1) Find a basis for $\ker(T)$.

(2) Find a basis for $R(T)$.

(b) Verify cayley-Hamiton theorem for the following matrix and hence, find A^{-1}. **07**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Q.4 **(a)** Find an orthogonal matrix P that diagonalizes **07**

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

(b) The vector space R^3 with the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectros **07**
$u_1 = (1, 1, 1), u_2 = (0, 1, 1), u_3 = (0, 0, 1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$.

OR

Q.4 **(a)** Define diagonalize of matrix A. Prove that if A is an $n \times n$ matrix, then the following are quivalent. **07**

(1) A is diagonalizable.

(2) A has a linearly independent eigenvectors.

(b) Find the minimal polynomial of **07**

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

Q.5 **(a)** Define the following terms : **07**

1. Orthogonal matrix
2. Proper and improper orthogonal matrix
3. Unitary matrix
4. Irreducible polynomial
5. Annihilating polynomial
6. Minimum polynomial

(b) Find the Jordan canonical forms of A **07**

$$\begin{bmatrix} -3 & 5 & 3 \\ -7 & 9 & 4 \\ 4 & -4 & 0 \end{bmatrix}$$

OR

Q.5 **(a)** Solve the following system by Gauss-Jordan elimination method: **07**
$x_1 + x_2 + 2x_3 = 8, -x_1 - 2x_2 + 3x_3 = 1, 3x_1 - 7x_2 + 4x_3 = 10$

(b) Solve the following equation using Crout't method **07**

$2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

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