Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY
ME SEMESTER II EXAMINATION – SUMMER 2017

Subject Code: 2720501 **Date: 25/05/2017**
Subject Name: Statistical Signal Analysis
Time:02:30 PM to 05:00 PM **Total Marks: 70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (1) There are n persons in a room. 04
(a) What is the probability that at least two persons have the same birthday?
(b) Calculate this probability for n = 50.
(c) How large need n be for this probability to be greater than 0.5?
(2) Explain different counting methods that aids in computing probabilities of statistical data. 03

(b) Let X be a continuous random variable with PDF 07

$$f_X(X) = \begin{cases} kx & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

(a) Determine the value of k and sketch $f_x(x)$.
(b) Find the mean and variance
(c) Find P(¼ < X < 2)

Q.2 (a) A binary source generates digits 1 and 0 randomly with probabilities 0.6 and 0.4, respectively. 07
(a) What is the probability that two 1s and three 0s will occur in a five-digit sequence?
(b) What is the probability that at least three 1s will occur in a five-digit sequence?

(b) The number of telephone calls arriving at a switchboard during any 10-minute period is known to be a Poisson r.v. X with λ= 2. 07
(a) Find the probability that more than three calls will arrive during any 10-minute period.
(b) Find the probability that no calls will arrive during any 10-minute period.

OR

(b) A WSS random process $X(t)$ with autocorrelation function 07

$$R_X(\tau) = e^{-\alpha|\tau|}$$

where α is a real positive constant, is applied to the input of an LTI system with impulse response

$$h(t) = e^{-bt}u(t)$$

where b is a real positive constant. Find the autocorrelation function of the output $Y(t)$ of the system.

Q.3 (a) (1) Let the random variable Y be defined by 04

$$Y = aX + b$$

where a is a nonzero constant. Suppose that X has cdf $F_X(x)$ then find $F_Y(y)$.
(2) Let a r.v. X denote the outcome of throwing a fair die. Find the mean and variance of X. 03

(b) (1.) Suppose that the number of particle emissions by a radioactive mass in t seconds is a Poisson random variable with mean λt. Use the Chebyshev inequality to obtain a bound for the 03 probability that $\left|\frac{N(t)}{t}-\lambda\right|$ exceeds ε.

(2) A fair coin is tossed 1000 times. Estimate the probability that the number of heads is between 400 and 600 (using the central limit theorem). Estimate the probability that the number is between 500 and 550. 04

OR

Q.3 (a) Let *X(t)* and *Y(t)* be both zero-mean and WSS random processes. Consider the random process 07
Z(t) defined by *Z(t)=X(t)+Y(t)*
(1) Determine the autocorrelation function and the power spectral density of *Z(t)*, (i) if *X(t)* and *Y(t)* are jointly WSS; (ii) if *X(t)* and *Y(t)* are orthogonal.
(2) Show that if *X(t)* and *Y(t)* are orthogonal, then the mean square of *Z(t)* is equal to the sum of the mean squares of *X(t)* and *Y(t).*

(b) Define and explain Ergodic process. Explain concept of ensemble average and time average. 07

Q.4 (a) Let *X, Y,* and *Z* be independent standard normal r.v.'s. Let $W = (X^2 + Y^2 + Z^2)^{1/2}$. Find the pdf of *W*. 07

(b) Consider a discrete-parameter random process *X(n)* = {X_n, $n \geq 1$) where the X_n' s are iid r.v.'s with common cdf $F_x(x)$, mean μ, and variance σ^2. 07
(1) Find the joint cdf of *X(n)*.
(2) Find the mean of *X(n)*.
(3) Find the autocorrelation function $R_X(n, m)$ of *X(n)*.
(4) Find the autocovariance function $K_X(n, m)$ of *X(n)*.

OR

Q.4 (a) The joint cdf of a bivariate r.v. (X, Y) is given by 07

$$F_{XY}(x,y) = \begin{cases} (1-e^{-\alpha x})(1-e^{-\beta x}), & x \geq 0, y \geq 0, \alpha, \beta > 0 \\ 0 & otherwise \end{cases}$$

(a) Find the marginal cdf's of *X* and *Y*.
(b) Show that *X* and *Y* are independent.
(c) Find *P(X ≤1, Y≤1), P(X ≤1), P(Y > 1).*

(b) (1) Compute the conditional means for given conditional pdfs. 04

$$f_{Y|X}(y|x) = \frac{1}{x} \qquad y \leq x < 1,\ \ 0 < x < 1$$

$$f_{X|Y}(x|y) = \frac{1}{1-y} \qquad y \leq x < 1,\ \ 0 < x < 1$$

(2) Define and explain (a) Gaussian Random Process (b) Strict Sense Stationary Process. 03

Q.5 (a) Inquiries arrive at a recorded message device according to Poison process of rate 15 inquiries per minute. Find the probability that in a 1-minute period, 3 inquiries arrive during the first 10 seconds and 2 inquiries arrive during the last 15 seconds. Find the mean and variance of the time until the arrival of the tenth inquiry 07

(b) (1) Show that 04

$$f_{XYZ}(x,y,z) = f_{Z|X,Y}(z|x,y) f_{Y|X}(y|x) f_X(x)$$

(2) Prove that covariance of independent random variable is zero. 03

OR

Q.5 (a) Two random processes *X(t)* and *Y(t)* are given by 07
$X(t) = A * cos(wt + \theta)$ and $Y(t) = A * cos(wt + \theta)$
where *A* and *w* are constants and *θ* is a uniform r.v. over (0, 2π). Find the cross-correlation function of *X(t)* and *Y(t)* and verify
Rx,y(-τ)=Rx,y(τ)

(b) Show that if a normal process is Wide Sense Stationary (WSS), then it is also strict-sense stationary. 07

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