Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME SEMESTER II EXAMINATION – SUMMER 2017

Subject Code: 2720714 **Date: 25/ 05 /2017**

Subject Name: MODERN CONTROL SYSTEMS

Time:02:30 PM to 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Represent the following differential equation in controllable canonical form and obseravable canonical form. **07**

$\dddot{y}$ +4$\ddot{y}$ + 5$\dot{y}$ +2y = 2$\ddot{u}$+$\dot{u}$ + 5$\dot{u}$ +2u, Where y and u are output and input of the system respectively.

(b) Explain significant of eigen values. Prove that the eigen values of a matrix are invariant under linear transformation. **07**

Q.2 **(a)** What is the controllability of the system? Derive the equaion for the state controllability and output controllablilty of the continuous time system. **07**

(b) Consider the following state equation and output equation : **07**

$$\begin{bmatrix} \dot{x1} \\ \dot{x2} \\ \dot{x3} \end{bmatrix} = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} + \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix} \mathrm{u} \quad ; \quad \mathrm{y}=[1 \quad 0 \quad 0] \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

Show that the state equation can be transformed into the following form by use of a proper transformation matrix: $\begin{bmatrix} \dot{z1} \\ \dot{z2} \\ \dot{z3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} z1 \\ 22 \\ 33 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$ then obtain the output y in terms of z_1, z_2, z_3.

OR

(b) Calculate the response of the system with a given initial condition of the state. **07**

$$\begin{bmatrix} \dot{x1} \\ \dot{x2} \\ \dot{x3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix} \; and \; x(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Q.3 **(a)** Determine the state feedback gain matrix k using Ackermann's formula of the the plant is given by **07**

$\begin{bmatrix} \dot{x1} \\ \dot{x2} \\ \dot{x3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, The system uses the state feedback control $u = -kx$, let us choose the desired closed loop poles at $s = -2 \pm j4 \; and \; s = -10.$.

(b) Explain the Krasovskii's theorem for the determination of asymptotic stability for the given closed loop control system. **07**

OR

Q.3 **(a)** Is the system completely state controllable and completely observable? **07**

$$\begin{bmatrix} \dot{x1} \\ \dot{x2} \\ \dot{x3} \end{bmatrix} = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \mathrm{u} \quad ; \quad \mathrm{y}=[1 \;\; 1 \;\; 0] \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

(b) Discuss the necessay and sufficent condition for arbitrary pole placement. Also discuss the direct subsitution method for determination of matrix K. **07**

Q.4 **(a)** Explain the describing functions method for non linear system stability analysis. **07**

(b) Define the Sylvester's criterion for checking the definiteness and also check the definiteness of the following functions **07**

$V(x) = x_1^2 + 2x_2^2$

$V(x) = 10x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_1x_3$

OR

Q.4 **(a)** Using the Lyapunov equation, determine the stability range of the gain K for the following state space model of the system. **07**

$$\begin{bmatrix} \dot{x1} \\ \dot{x2} \\ \dot{x3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 1 \\ -K & 0 & -1 \end{bmatrix} - \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} r,$$

(b) Explain how the Phase plane method is useful in determining the behaviour of Nonlinear systems. **07**

Q.5 **(a)** Consider the system **07**

$A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [0 \;\; 1]$, design a full order state observer, assume that the desired eigen values of the observer matrix are $\mu_1 = -10, \mu_2 = -10$

(b) Develop the block diagram of observed state feedback control system and derive the transfer function of the observer based controller. **07**

OR

Q.5 **(a)** Determine the optimal feedback gain matrix K such that the following performance index is minimized: **07**

$J = \int_0^\infty (X^T Q X + u^2) dt$ where $Q = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix}$ $(\mu \geq 0)$, the state equation of the plant $\dot{x} = Ax + Bu \;\; where, A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \;\; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) Derive the reduced matrix Riccati equation for quadratic optimal regulator problem. **07**

****