Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E. SEMESTER II–EXAMINATION Summer 2017

Subject code: 2724712 **Date:31/05/2017**

Subject Name: Optimization Theory and Practice

Time: 02:30 p.m.-5:00 p.m. **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** A manufacturer produces two types of machine parts, P_1 and P_2, using lathes and milling machines. The machining times required by each part on the lathe and the milling machine and the profit per unit of each part are given below: **07**

Machine	Machine Time Required: Lathe	Machine Time Required: Milling Machine	Profit Per Unit
P_1	5	2	200
P_2	4	4	300

If the total machining times available in a week are 500 hours on lathes and 400 hours on milling machines, determine the number of units of P_1 and P_2 to be produced per week to maximize the profit by Simplex Method.

(b) (i) Explain the terms: Chromosome, Population, Fitness Function **03**

(ii) Describe Particle Swarm Optimization algorithm. **04**

Q.2 **(a)** Use Fletcher-Reeves method to minimize the function **07**

$$f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$$

starting from the point $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

(b) Minimize $F = x_1 + 2x_2 + 3x_3$ **07**

subject to

$$x_1 - x_2 + x_3 \geq 4, \qquad x_1 + x_2 + 2x_3 \leq 8, \qquad x_2 - x_3 \geq 2$$
$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0$$

by Dual Simplex Method.

OR

(b) Use Graphical Method to solve the following problem: **07**

Maximize $f = 2x_1 + 6x_2$

subject to

$$-x_1 + x_2 \leq 1, \qquad 2x_1 + x_2 \leq 2,$$
$$x_1 \geq 0, \qquad x_2 \geq 0.$$

Q.3 **(a)** (i) Find the points at which the function **03**

$$y = 2x^3 - 3x^2 - 12x + 4$$

has maximum and minimum values.

(ii) Write some engineering applications of optimization. Describe one of them in detail. **04**

(b) (i) Consider the following optimization problem: **03**

$$Minimize\ f(X) = x_1 + x_2$$

subject to

$$x_1^2 + x_2 \geq 2, \qquad 4 \leq x_1 + 3x_2, \qquad x_1 + x_2^4 \leq 30$$

Use Kuhn-Tucker conditions to check whether $\boldsymbol{X} = [1, 1]^T$ is a local minimum. What are the values of the Lagrange multipliers?

(ii) Find the extreme points of the function **04**

$$f(x, y) = x^2 + y^2 + xy + x - 4y + 5.$$

OR

Q.3 **(a)** (i) How gradient vector and Hessian matrix for a function of n variables are useful for optimization of the function? **03**

(ii) Explain the terms: Feasible Region, Unimodal Function **04**

(b) (i) Determine whether the following matrix is positive definite, negative definite or indefinite by evaluating the signs of its sub-matrices: **03**

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}.$$

(ii) Maximize $f(x_1, x_2) = \pi x_1^2 x_2$ subject to $2\pi x_1^2 + 2\pi x_1 x_2 = 24\pi$ using Lagrange's multiplier method. **04**

Q.4 **(a)** Use Dichotomous Search Method to find the minimum of the function $f(x) = x(x - 1.5)$ in the interval $(0, 1)$ to within 10 % of the exact value. Take $\delta = 0.001$. **07**

(b) Find the minimum of the function **07**

$$f(\lambda) = 0.65 - \frac{0.75}{1 + \lambda^2} - 0.65\, \lambda\, tan^{-1} \frac{1}{\lambda}$$

using quasi-Newton method with the starting point $\lambda_1 = 0.1$ and the step size $\Delta\lambda = 0.01$. Use $\varepsilon = 0.01$ for checking the convergence.

OR

Q.4 **(a)** Minimize the function $f(x) = x^3 - 4x$ in the interval $(0.5, 2.0)$ using the Fibonacci Method with $n = 6$. **07**

(b) Find the minimum of the function $f(\lambda) = \lambda^5 + 5\lambda^3 - 20\lambda + 10$ using Newton Method with the starting point $\lambda_1 = 0.5$. Take $\varepsilon = 0.01$. **07**

Q.5 **(a)** What are the advantages of random search method? Also describe Random Jumping Method. **07**

(b) Describe the basic approach of the penalty function method. **07**

OR

Q.5 **(a)** (i) Mention the transformations used for converting a constrained optimization problem into an unconstrained one. **03**

(ii) Perform two iterations of Cauchy's steepest descent method to minimize the function **04**

$$f(x_1, x_2) = 2x_1^2 + 2x_1 x_2 + x_2^2 + x_1 - x_2$$

by taking $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ as the starting point.

(b) Write the Sequential Linear Programming Algorithm. **07**

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