Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY
ME SEMESTER – I (OLD) EXAMINATION – SUMMER 2017

Subject Code: 710401N **Date:08/05/2017**
Subject Name: Statistical Signal Analysis
Time:02:30 P.M. to 05:00 P.M. **Total Marks: 70**
Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define Cumulative Distribution Function. State and prove all the properties of CDF. **07**

(b) What do you mean by conditional probability? What is conditional expectation? Determine E[E[Y|X]]. **07**

Q.2 **(a)** Define characteristic function. Find the characteristic function of the uniform random variable in the interval [a,b]. Also find the characteristic function of exponential random variable with PDF $f_X(x) = \lambda e^{-\lambda x}$, $x \geq 0$ and $\lambda > 0$. **07**

(b) State and prove bayes' rule. Two numbers x and y are selected at random between 0 and 1. Let the events A and B be defined as A = {$x > 0.5$} and B = {y > 0.5}. Are the events A and B independent? **07**

OR

(b) Define the Probability Density Function. Determine the mean, the mean square value and the variance of the RV X whose PDF is given by $p_x(x) = 0.5|x|e^{-|x|}$. **07**

Q.3 **(a)** Show that the poisson distribution can be used as a convenient approximation to the binomial distribution for large n and small p. **07**

(b) Let the random variable Y be defined by Y=aX+b, where a is a nonzero constant. Suppose that X has cdf $F_X(x)$, then find $F_Y(y)$ and $f_Y(y)$ **07**

OR

Q.3 **(a)** State and prove Chebyshev inequality. Show that chebyshev's inequality is useful to decide the width of the PDF. **07**

(b) Let X be a continuous random variable with PDF **07**

$$f_X(x) = \begin{cases} kx & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

(a) Determine the value of k and sketch $f_X(x)$

(b) Find and sketch corresponding CDF $F_X(x)$

(c) Find $P(1/4 < X \leq 2)$.

Q.4 **(a)** The joint CDF is given by: **07**

$$F_{X,Y}(x,y) = \begin{cases} (1 - e^{-\alpha x})(1 - e^{-\beta y}) & x \geq 0, y \geq 0 \\ 0 & elsewhere \end{cases}$$

Find the marginal CDF's. Find the probability of the events A = {X≤1, Y≤1} and B={X>x, Y>y}.

(b) State and prove central limit theorem. **07**

OR

Q.4 **(a)** What is the law of large numbers? Explain strong and weak law of large numbers with example. **07**

(b) What is convergence of random variable? Explain sure convergence, almost sure covergence and mean square convergence with an example. **07**

Q.5 **(a)** Classify the random processes and explain each in detail. **07**

(b) Sketch the ensemble of the random process $x(t) = A \cos(\omega t + \Theta)$, where A and ω are constants and Θ is an R.V. uniformly distributed in the range $(0, 2\pi)$. Just by observing the ensemble, determine if this is a stationary or nonstationary process. Also determine whether this is a wide-sense stationary process. **07**

OR

Q.5 **(a)** Explain the concept of mean square derivatives of a random process with necessary equations. **07**

(b) Determine the PSD and the mean square value of a random process $x(t) = A \cos(\omega t + \Theta)$, where A and ω are constants and Θ is an R.V. uniformly distributed in the interval $(0, 2\pi)$. **07**

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