Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME SEMESTER – I (OLD) EXAMINATION – SUMMER 2017

Subject Code: 710703N **Date:10/05/2017**

Subject Name: MODERN CONTROL SYSTEMS

Time:02:30 P.M. to 05:00 P.M. **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Derive the state space model of speed control of DC motor using armature controlled method. **07**

(b) **07**

Define (1) State Variable (2)State Space (3) State Vector (4) State Equation (5) State Model (6) Controllability (7) Observability

Q.2 **(a)** Obtain the transfer function of the system defined by **07**

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix}=\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u \;;\quad y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

(b) What is Jordan Canonical form ? Discuss its properties. **07**

OR

(b) Consider the matrix $A=\begin{bmatrix}0 & 1\\ 0 & -2\end{bmatrix}$, compute e^{At} using sylvester's interpolation formula. **07**

Q.3 **(a)** Define controllability of the system. Is the following system completely state controllable. **07**

$$\begin{bmatrix}\dot{x1}\\ \dot{x2}\\ \dot{x3}\end{bmatrix}=\begin{bmatrix}-1 & -2 & -2\\ 0 & -1 & 1\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}x1\\ x2\\ x3\end{bmatrix}+\begin{bmatrix}2\\ 0\\ 1\end{bmatrix}u \quad ;\quad y=\begin{bmatrix}1 & 1 & 0\end{bmatrix}\begin{bmatrix}x1\\ x2\\ x3\end{bmatrix}$$

(b) What is State Transition matrix? How it is obtained? Discuss its properties **07**

OR

Q.3 **(a)** Explain eigen values and eigenvectors of a matrix. Hence, prove that the Eigen values of a matrix are invariant under linear transformation **07**

(b) Is the following system completley observable? **07**

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u;\quad y=\begin{bmatrix}20 & 9 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Q.4 **(a)** Explain variable gradient method to construct Liapunov function. **07**

(b) Consider the system described by the equations **07**

$\dot{x_1}=-x_1-x_2^2$ and $\dot{x_2}=-x_2$. Investigate the stability of the equilibrium state. Use direct method of Lyapunov.

OR

Q.4 **(a)** Using the Lyapunov equation, determine the stability range of the gain K for the following state space model of the system. **07**

$$\begin{bmatrix}\dot{x1}\\ \dot{x2}\\ \dot{x3}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & -2 & 1\\ -K & 0 & -1\end{bmatrix}-\begin{bmatrix}x1\\ x2\\ x3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ K\end{bmatrix}r,$$

(b) Explain the principle of duality that clarify apparent analogy between controllability and observability. **07**

Q.5 **(a)** Consider the regulator system, the plant is given by **07**

$$\begin{bmatrix} \dot{x1} \\ \dot{x2} \\ \dot{x3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$, The system uses the state feedback control $u = -kx$, let us choose the desired closed loop poles at $s = -2 \pm j4 \; and \; s = -10$. Determine the state feedback gain matrix k.

(b) What is the pole placement? Derive the equation for Matrix K using Ackermann's formula. **07**

OR

Q.5 **(a)** Compare classical control theory with the modern control theory. **07**

(b) With neat block diagram representations discuss cascade decomposition method. **07**