## **GUJARAT TECHNOLOGICAL UNIVERSITY** M. E. - SEMESTER – II • EXAMINATION – SUMMER • 2013

Subject code: 1722309 Subject Name: Numerical Methods Time: 10.30 am – 01.00 pm Instructions: Date: 07-06-2013

**Total Marks: 70** 

- 1. Attempt all questions.
- 2. Make suitable assumptions wherever necessary.
- 3. Figures to the right indicate full marks.
- Q.1 (a) (i) Find a real root of the equation  $x^3 + 4x^2 1 = 0$  in the interval 03 [0,1] by using Bisection method. Perform four iterations.
  - (ii) Find a real root of the equation 3x = cosx + 1 correct to three decimal places by using Newton-Raphson Method. Take  $x_0 = 0$ .
  - (b) Derive the formula for generating approximations of the root by 07 using False Position Method and hence find a root of the equation  $xe^{x} cosx = 0$

using False Position Method correct up to  $\varepsilon_a < 10$  % in [0, 1].

Q.2 (a) A total charge Q is uniformly distributed around a ring-shaped 07 conductor with radius a. A charge q is located at a distance x from the center of the ring. The force exerted on the charge by the ring is given by

$$F = \frac{1}{4\pi e_0} \frac{qQx}{(x^2 + a^2)^{\frac{3}{2}}}$$

where  $e_0 = 8.85 \times 10^{-12} C^2 / (Nm^2)$ . Find the distance x where the force is 1.25 N if q and Q are  $2 \times 10^{-5}$  for a ring with radius 0.9 m using Newton-Raphson Method.

(b) What is mathematical modeling? Develop a model to find the velocity 07
 v of a freely falling parachutist as a function of time t assuming that the air resistance is linearly proportional to v. How can you solve the problem numerically?

## OR

## (b) Describe the Gauss Elimination Algorithm. 07

Q.3 (a) (i) Solve the following system of equations by Gauss Elimination 03 Method:

x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40(ii) Solve the following system of equations using Gauss Seidel 04

Method using initial guess 
$$x_0 = y_0 = z_0 = 0$$
:  
 $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$ 

(b) Use matrix Inversion Method to find the solution of the system:  

$$x + y + 2z = 4, x + 2y + 3z = 8, 2x + 3y + z = 12.$$
07

OR

Q.3 (a) Describe in detail the pitfalls of Gauss Elimination Method. Also 07 explain how you will detect a singular system.

(b) The following system of equations was generated by applying the 07mesh current law to a circuit:

 $60I_1 - 40I_2 = 200,$  $40\bar{I}_1 + 15\bar{0}I_2 - 10\bar{0}I_3 = 0,$  $-100I_2 + 130I_3 = 230$ Solve for  $I_1$ ,  $I_2$  and  $I_3$ .

Q.5

Q.5

(a) (i) The following data was taken from an experiment that measured 05 Q.4 the current in a wire for various imposed voltages:

| V,V | 2   | 3 4 |      | 5  | 7    | 10   |  |
|-----|-----|-----|------|----|------|------|--|
| i,A | 5.2 | 7.8 | 10.7 | 13 | 19.3 | 27.5 |  |

On the basis of a linear regression of this data, determine current for a voltage of 3.5 V.

- 02 (ii) What is the difference between the two methods of curve fitting, namely, regression and interpolation?
- (b) Give the names of three of the nonlinear curves which can be 07 transformed to a linear equation. Describe the procedure to fit such nonlinear curves to the given data.

OR

- (a) Find the equation of a straight line,  $y = a_0 + a_1 x$  which best fits with **Q.4** 07 the given points  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  using least squares method.
  - (b) (i) Apply Newtonøs backward difference formula to the data below to 03 obtain a polynomial of degree 4 in x:

| x | 1 | 2  | 3 | 4  | 5 |
|---|---|----|---|----|---|
| y | 1 | -1 | 1 | -1 | 1 |

|            | (ii) Use Newtonøs divided difference formula to evaluate $f(8)$ given                   |                   |          |         |        |         |          |          | 04   |    |
|------------|---|-------------------|----------|---------|--------|---------|----------|----------|------|----|
|            |   | x                 | 4        | 5       | 7      | 10      | 1        | 1 1      | 3    |    |
|            |   | f(x)              | 48       | 100     | 294    | 900     | ) 12     | 10 20    | 28   |    |
| <b>(a)</b> | ) (i) The following table gives the velocity $v$ of a particle at time $t$ :            |                   |          |         |        |         |          | 03       |      |    |
|            |   | t (sec)           | 0        | 2       | 4      | 6       | 8        | 10       | 12   |    |
|            |   | <b>v (</b> m/sec) | 4        | 6       | 16     | 34      | 60       | 94       | 136  |    |
|            | Find the distance moved by the particle in 12 seconds.                                  |                   |          |         |        |         |          |          |      |    |
|            | (ii) Write the algorithm of the Lagrange interpolation method.                          |                   |          |         |        |         |          |          | 04   |    |
| (b)        | Apply Runge-Kutta method of order 4 to find approximate value of y                      |                   |          |         |        |         |          |          |      |    |
|            | for $x = 0.2$ , in steps of 0.1, if $\frac{dy}{dx} = x + y^2$ , given that $y = 1$ when |                   |          |         |        |         |          |          | l    |    |
|            | <u>x</u> –  | 0.                |          |         |        |         |          |          |      |    |
|            |   |                   |          |         | OR     |         |          |          |      |    |
| <b>(a)</b> | A st  | teady state h     | eat bala | nce for | a 10-m | rod car | n be rep | oresente | d as | 07 |
|            | $d^2T$  | രദ്ണം –           |          |         |        |         |          |          |      |    |
|            |   | -0.15T =          | ÷ U      |         |        |         |          |          |      |    |

- $dx^2 = 0.151$ with T(0) = 240 and T(10) = 150. Use the finite-difference approach with  $\Delta x = 2 m$  to solve the given boundary value problem.
- (b) (i) Why do we need the methods for numerical integration? 03 (ii) Describe the algorithm of Euler Method. 04

\*\*\*\*\*