

GUJARAT TECHNOLOGICAL UNIVERSITY**M. E. - SEMESTER – I • EXAMINATION – SUMMER • 2013****Subject code: 710107****Date: 11-06-2013****Subject Name: Quantum Theory & Algorithm Design****Time: 10.30 am – 01.00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Define the following terms 07

- (1) Basis
- (2) Normal Operator
- (3) Unitary Operator
- (4) Hermitian Operator
- (5) Projection Operator
- (6) Positive Operator
- (7) Density Operator

- (b) (1) Define: Countably infinite set 07**
 (2) Prove that an equivalence relation is the same as a partition
 (3) Find the inverse of the function,

$$f(n) = (-1)^n \lceil n/2 \rceil$$

Q.2 (a) Construct an orthonormal basis set using Gram Schmidt process from, 07

$$|v_1\rangle = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, |v_2\rangle = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, |v_3\rangle = \begin{pmatrix} 3 \\ -7 \\ 1 \end{pmatrix}$$

- (b) What is pivot? Find LU decomposition of matrix, 07**

$$\begin{pmatrix} 2 & 3 & 1 & 5 \\ 6 & 13 & 5 & 19 \\ 2 & 19 & 10 & 23 \\ 4 & 10 & 11 & 31 \end{pmatrix}$$

OR

- (b) What is Schur complement? Find LUP decomposition of matrix, 07**

$$\begin{pmatrix} 2 & 0 & 2 & 0.6 \\ 3 & 3 & 4 & -2 \\ 5 & 5 & 4 & 2 \\ -1 & -2 & 3.4 & -1 \end{pmatrix}$$

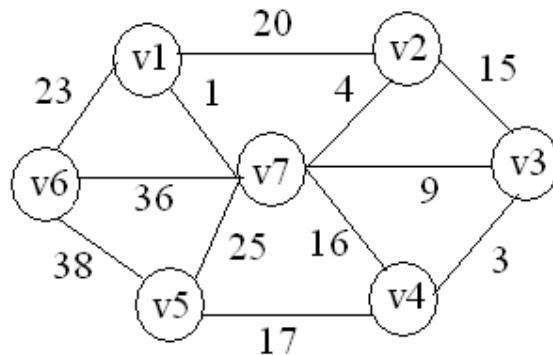
Q.3 (a) Find the eigen values and eigen vectors for the $\tilde{\sigma}_y$ gate, which has the matrix representation 07

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

- (b) What is an Amortized analysis? Explain Merge sort using divide and conquer and also make comments on complexity analysis for merge sort. 07

OR

- Q.3 (a) Explain Commutator Algebra and Unitary Transformation and Show that $[\sigma_1, \sigma_2] = 2i \sigma_3$ 07
- (b) Define: Spanning Tree. Find out minimum cost of spanning tree by using kruskal algorithms for the following graph 07



- Q.4 (a) Find $X \otimes Z |\psi\rangle$ 07
where,
 $|\psi\rangle = \frac{|0\rangle|0\rangle - |1\rangle|1\rangle}{\sqrt{2}}$

And also find the tensor product of Pauli matrices X and Z

- (b) What is Feasible solution? What is the difference between Greedy and Dynamic Programming? Solve the following knapsack problem using Greedy method. Capacity of knapsack is 20 07
- | Objects | Profits | Weights |
|---------|---------|---------|
| 1 | 25 | 18 |
| 2 | 24 | 15 |
| 3 | 15 | 10 |

OR

- Q.4 (a) Explain Bloch vector. Prove that for a mixed state, $\text{Tr}(\rho^2) < 1$ 07
- Q.4 (b) What is an optimal solution? What is the difference between Divide - Conquer and Dynamic Programming? Solve the following knapsack 0/1 problem using dynamic programming. Capacity of the knapsack is 6 07
- | Objects | Profits | Weights |
|---------|---------|---------|
| X1 | 1 | 2 |
| X2 | 2 | 3 |
| X3 | 5 | 4 |

Q.5 (a) A system is found to be in the state

07

$$|\psi\rangle = \frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$$

- (1) Write down the density operator for this state
- (2) Show that this is a pure state
- (3) Find $\langle X \rangle$

(b) Draw Huffman tree for the frequencies 6,2,3,3,4,9 and explain Euler tour traversal binary tree algorithm **07**

OR

Q.5 (a) Suppose

07

$$|a\rangle = \frac{1}{\sqrt{3}}|+\rangle + \frac{\sqrt{2}}{\sqrt{3}}|-\rangle$$
$$|b\rangle = \frac{2}{3}|+\rangle - \frac{\sqrt{5}}{9}|-\rangle$$

with 75% of the system prepared in the state $|a\rangle$ and 25% of the system prepared in the state $|b\rangle$

- (1) Write down the density operators ρ_a and ρ_b
- (2) Compute the density operator for the ensemble
- (3) If a measurement is made, What is the probability of finding $|+\rangle$?

(b) Design and Explain Dijkstra's shortest path algorithm

07
