Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – III • EXAMINATION – SUMMER • 2013

Subject code: 730403 **Date: 15-05-2013**

Subject Name: Optimization Techniques

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find the maxima and minima, if any, of the function **03**
$f(x) = 64x^3 - 144x^2 + 108x - 27.$

(ii) Find the extreme points of the function **04**
$f(x_1, x_2) = x_1^2 + x_2^3 + 2x_1^2 + 4x_2^2 + 6.$

(b) Find the dimensions of a closed cylindrical tin made up of sheet metal to maximize its volume such that the total surface area is 24π using Lagrange's method of multipliers. **07**

Q.2 **(a)** Consider the following problem: **07**
Minimize $f(X) = x_1^2 + x_2^2 + x_3^2$
subject to $x_1 + x_2 + x_3 \geq 5,\ \ 2 - x_2 x_3 \leq 0,$
$x_1 \geq 0,\ \ x_2 \geq 0,\ \ x_3 \geq 2.$
Determine whether the Kuhn-Tucker conditions are satisfied at the following points:
$X_1 = \left[\frac{3}{2}, \frac{3}{2}, 2\right]^T,\qquad X_2 = \left[\frac{4}{3}, \frac{2}{3}, 3\right]^T,\qquad X_3 = [2, 1, 1]^T$

(b) Use Simplex Method to maximize $f = x_1 + 2x_2 + x_3$ **07**
subject to
$2x_1 + x_2 - x_3 \leq 2,\ \ 3x_1 - x_2 + 5x_3 \leq 6,$
$4x_1 + x_2 + x_3 \leq 6,\qquad x_i \geq 0,\ \ i = 1, 2, 3.$

OR

(b) Use Revised Simplex Method to maximize $f = 2x_1 + x_2$ **07**
subject to $3x_1 + 4x_2 \leq 6,\ \ 6x_1 + x_2 \leq 3,\ \ x_1 \geq 0,\ x_2 \geq 0.$

Q.3 **(a)** Use Dichotomous Search Method to find the minimum of **07**
$f = x(x - 1.5)$ in the interval $(0.0, 1.00)$ to within 10% of the exact value. Take $\delta = 0.001.$

(b) Use Newton's Method to minimize the function **07**

$$f(\lambda) = 3\lambda^5 - 10\lambda^3 - 45\lambda + 1$$

with $\lambda_1 = 3.$ Iterate until $|f'(\lambda)| < 0.05.$

OR

Q.3 **(a)** Use Fibonacci Method with $n = 6$ to minimize the function **07**

$$f(x) = 0.65 - \frac{0.75}{1 + x^2} - 0.65\, x \tan^{-1}\left(\frac{1}{x}\right)$$

in the interval $[0, 3].$

(b) Use Secant Method to find the minimum of $f = x(x - 3)$ with an initial step size of $t_0 = 0.2, \lambda_1 = 0.0,$ and $\varepsilon = 0.01.$ **07**

Q.4 **(a)** Perform three iterations of Interval Halving Method to minimize the function $f(\lambda) = \lambda^5 - 5\lambda^3 - 20\lambda + 5$ in the interval $(0, 5).$ **07**

(b) Write the algorithm of Hooke and Jeeves' Method. **07**

OR

Q.4 **(a)** Use Golden Section Method to minimize the function **07**

$$f(\lambda) = \frac{\lambda}{\log \lambda}$$

with $n = 6$ in the interval $(0,3)$.

(b) Write the algorithm of Random Walk Method. **07**

Q.5 **(a)** Use Fletcher-Reeves method to minimize **07**

$$f(x_1,x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$$

from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform only two iterations.

(b) Write the algorithm of Sequential Linear Programming for minimization of a constrained multi-variable non-linear optimization problem. **07**

OR

Q.5 **(a)** Minimize $f(x_1,x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ from the starting point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ using Powell's Method. Take $\varepsilon = 0.01$. **07**

(b) Describe the basic approach of the penalty function method. Also write the algorithm of Interior Penalty function Method. **07**

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