Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – II • EXAMINATION – SUMMER • 2014

Subject code: 1722309 **Date: 25-06-2014**

Subject Name: Numerical Methods

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Find a real root of the equation $x^{3.5} = 80$ in the interval [3,4] by using Bisection method. Perform four iterations. **03**

(ii) Find a real root of the equation $8\, sinx\, e^{-x} = 1$ correct to three decimal places by using False Position method with $x_0 = 0.1$, $x_1 = 0.3$. **04**

(b) What are the pitfalls of Newton-Raphson Method? Find the fourth root of 30 using Newton-Raphson method. Perform only three iterations. **07**

Q.2 **(a)** Real mechanical systems may involve the deflection of nonlinear springs. If a mass m is released a distance h above a nonlinear spring, the resistance force F of the spring is given by **07**

$$F = -(k_1 d + k_2 d^{3/2})$$

Conservation of energy can be used to show that

$$0 = \frac{2k_2 d^{5/2}}{5} + \frac{1}{2}k_1 d^2 - mgd - mgh.$$

Solve for d, given the following parameter values: $k_1 = 50{,}000\ g/s^2$, $k_2 = 40\ g/(s^2 m^{0.5})$, $m = 90\ g$, $g = 9.81\ m/s^2$, and $h = 0.45\ n$

(b) Write the algorithm of the Bisection Method. **07**

OR

(b) Describe different types of errors. What are true percentage relative error and approximate relative percentage error? **07**

Q.3 **(a)** (i) Solve by Gauss Elimination Method: **03**

$$-x + y + z = -1,\ 2x + 3y + 4z = 3,\quad 4x - 5y + 6z = 15$$

(ii) Find the inverse of the matrix using Gauss Jordan Method: **04**

$$\begin{bmatrix} 2 & 1 & -1 \\ 5 & 2 & 2 \\ 3 & 1 & 1 \end{bmatrix}.$$

(b) Check whether the following system of equations is diagonally dominant or not. If not, arrange the equations so that it becomes diagonally dominant and then solve it using Gauss Seidel Method using initial guess: $x_0 = y_0 = z_0 = 0$: **07**

$2x - 6y - z = -38$, $-3x - y + 7z = -34$, $-8x + y - 2z = -20$

OR

Q.3 **(a)** Write the algorithm of Gauss Seidel Method. **07**

(b) Idealized spring-mass systems have numerous applications throughout engineering. For an arrangement of four springs in series depressed with a **07**

force of 1500 kg, the force balance equations at equilibrium are:

$$k_2(x_2 - x_1) = k_1 x_1, \qquad k_3(x_3 - x_2) = k_2(x_2 - x_1),$$
$$k_4(x_4 - x_3) = k_3(x_3 - x_2), \qquad F = k_4(x_4 - x_3)$$

where the kôs are spring constants. If k1 through k4 are 100, 50, 80, and 200 N/m respectively, compute the xôs.

Q.4 **(a)** (i) The result of measurement of electric resistance ⍰ of a copper bar at various temperatures $t°C$ are listed below: **05**

⍰	19	25	30	36	40	45	50
⍰	76	77	79	80	82	83	85

Find a relation R = a + bt and determine R for t = 20

(ii) The current in a wire is measured with great precision as a function of time: **02**

⍰	0	0.1250	0.2500	0.3750	0.5000
⍰	0	6.24	7.75	4.85	0.0000

Determine i at $t = 0.45$.

(b) The population (p) of a small community on the outskirts of a city grows rapidly over a 20-year period: **07**

⍰	0	5	10	15	20
⍰	100	200	450	950	2000

As an engineer working for a utility company, you must forecast the population 5 years into the future in order to anticipate the demand for power. Employ an exponential model to make this prediction.

OR

Q.4 **(a)** Explain in detail the complete procedure to find the coefficients of the equation of a parabola, $y = a_0 + a_1 x + a_2 x^2$,which best fits with the given points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ using least squares method. **07**

(b) (i) Apply Newtonôs forward difference formula to the data below to obtain a polynomial of degree 3 in x : **03**

⍰	4	6	8	10
⍰	1	3	8	16

Hence evaluate y for x= 5

(ii) Use Newtonôs divided difference formula to find the value of log_{10}^{656} if it is given that $log_{10}^{654} = 2.8156$, $log_{10}^{658} = 2.8182$, $log_{10}^{659} = 2.8189$, $log_{10}^{661} = 2.8202$. **04**

Q.5 **(a)** (i) Given the data below, find the isothermal work done on the gas as it is compressed from 23 L to 3 L from the following data using Trapezoidal rule: **03**

⍰, ⍰	3	8	13	18	23
⍰, ⍰⍰⍰	12.5	3.5	1.8	1.4	1.2

Use the formula $W = -\int_{V_1}^{V_2} P\, dV$.

(ii) Write the algorithm to fit the power equation $y = ax^b$ to the given points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ using least squares method. **04**

(b) Apply Runge-Kutta method of order 4 to find approximate value of ⍰ for $x = 0.2$, in steps of 0.1, if **07**

$$\frac{dy}{dx} = \frac{y - x}{y + x},$$

given that Y = 1when X= 0

OR

Q.5 **(a)** Use finite-difference approach with $\Delta x = 2$ to solve the boundary value problem **07**

$$7\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y + x = 0$$

with y(0) = 5 and y(10) = 8

(b) (i) What is the importance of numerical methods? **03**

(ii) Write the algorithm of Heunôs Method. **04**

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