Enrolment No.

## GUJARAT TECHNOLOGICAL UNIVERSITY M. E. - SEMESTER – I • EXAMINATION – SUMMER • 2014

| M. E SEMESTER – I • EXAMINATION – SUMMER • 2014 |            |   |          |
|---|------------|---|----------|
| Subject code: 710401NDate: 13-06-2              |            |   |          |
| Subject Name: Statistical Signal Analysis       |            |   |          |
| Time: 02:30 pm - 05:00 pm Total Marks: 70       |            |   |          |
| Instructions:                                   |            |   |          |
|   |            | Attempt all questions.<br>Make suitable assumptions wherever necessary.   |          |
|   |            | Figures to the right indicate full marks.   |          |
| Q.1   | (a)        | Define CDF. State and prove its properties.   | 07       |
| Q.1   | (a)<br>(b) | (i). Explain Total probability and Bayeøs theorem   | 04       |
|   | ( )        | (ii). The probability of a bit error in communication line is $10^{-3}$ . Find the  |          |
|   |            | probability that a block of 1000 bits has five or more errors.  | 03       |
| Q.2   | (a)        | <ul><li>A company producing electric relays has three manufacturing plants A, B and C producing 50, 30, and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.</li><li>(i). If a relay is selected at random from the output of the company, what is the probability that it is defective?</li></ul> |          |
|   |            | <ul><li>(ii). If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant B?</li></ul>  | 07       |
|   | (b)        | (i). Let $Y = a \cos(t + \cdot)$ where a, and t are constants and is a uniform random variable in the interval (0,2). The random variable Y results from sampling the amplitude of a sinusoid with random phase . Find the expected   | 0.4      |
|   |            | values of Y and expected value of power of Y.<br>(ii). Find the variance of the random variable X that is uniformly distributed in the interval [a, b].   | 04<br>03 |
|   |            | OR  |          |
|   | (b)        | Give answer of following questions.   | 0.4      |
|   |            | <ul><li>(i). Discuss statistical independence and uncorrelation of random variables.</li><li>(ii). Give the expression for the PDF of a Gaussian random variable and show</li></ul>   | 04       |
|   |            | that Gaussian PDF integrates to one.  | 03       |
| Q.3   | (a)        | State and explain Markov and Chebysheves inequalities.  | 07       |
|   | (b)        | Find the normalization constant c and the marginal PDF¢s for the following joint PDF :  |          |
|   |            | $f_{X,Y}(x,y) = ce^{-x}e^{-y}$ , 0 Ö y Ö x ÖÔ<br>= 0 , elsewhere  |          |
|   |            | Also, Find P[ $X + Y \ddot{O}1$ ].  | 07       |
|   |            | OR  |          |
| Q.3   | <b>(a)</b> | State and prove Central limit theorem.  | 07       |
|   | (b)        | Let the random variable Y be defined by $Y = aX + b$ , Where a is a nonzero constant. Suppose that X has CDF $F_X(x)$ , then find $F_Y(y)$ and $f_Y(y)$ .   | 07       |
| Q.4   | (a)        | For random process define cross correlation function and cross power spectral density. Give useful property of cross power spectral density.  | 07       |
|   |            |   |          |

(b) The joint CDF for the vector of random variables X = (X, Y) is given by  $F_{X,Y}(x,y) = (1 \circ e^{\circ x}) (1 - e^{-y})$ ,  $x \times 0, y \times 0$  = 0, elsewhere Find the marginal CDFøs. Find the probability of events  $A = \{ X \ddot{O}1, Y \ddot{O}1 \}, B = \{X > x, Y > y\}.$ 

- Q.4 (a) What is convergence of Random variable? Explain sure convergence, Almost sure convergence.
  - (b) A random telegraph signal is passed through a RC low pass filter which has a Transfer function

$$H(f) = \frac{\beta}{\beta + j2\pi f}$$

Where = 1/RC is the time constant of the filter. The Auto correlation function of the Telegraph signal is  $R_X() = 1 - e^{-2||}$ . Find the power spectral density and the autocorrelation of the output.

Q.5 (i). Define characteristics function and Moment generating function of random (a) variable. 03 (ii). For a random process X(t), give the definitions of Mean, Autocorrelation Auto covariance and Correlation coefficient. 04 (b) Consider a random amplitude sinusoid signal with period T,  $X(t) = A \cos(2 t/T)$ Is X(t) cyclostationary ? Wide sense cyclostationary ? 07 OR Q.5 Give the answer of following questions. **(a)** (i). Are the wiener and Poisson processes mean square continuous? 03 (ii). Does the Wiener process whose autocorrelation function is given by  $\min(t_1, t_2)$  have a mean square derivative? What is the name of the  $R_{x}(t_{1},t_{2}) =$ process obtained by taking derivative of Wiener process? 04 (b) Explain Mean square convergence and convergence in probability with an example. 07

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