Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – SUMMER • 2014

Subject code: 710901N **Date: 13-06-2014**

Subject Name: Theory of Elasticity

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** The correct strains of strained continuum must be compatible. Justify the statement by deriving six compatability conditions as regards to the material continuity. **07**

(b) (i) Consider a stress function ϕ (x, y) and σ_x, σ_y and τ_{xy} are given below. Show that these values satisfy the differential equation of equilibrium in absence of the body force. **03**

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2},\ \sigma_y = \frac{\partial^2 \phi}{\partial x^2}, \tau_{xy} = \frac{\partial^2 \phi}{\partial x \partial y}$$

(ii) If the stress components are directly proportional to the corresponding strain components, the values of the principal stresses are related to the values of the principal strains by the same proportional factor. Evaluate. **04**

Q.2 **(a)** For the state of stress given below **07**

$$\begin{bmatrix} 10 & 4 & 6 \\ 4 & 2 & 8 \\ 6 & 8 & 6 \end{bmatrix}$$

determine normal and shearing stress on octahedral plane. Compare these with the τ_{oct} and σ_{oct} calculated for the hydrostatic and pure shear states. Are the octahedral planes for the given state, the hydrostatic and pure shear state the same or different? Validate your answer.

(b) An element in plane stress is subjected to stresses σ_{xx} = 15 kN/cm^2, σ_{yy} = 5 kN/cm^{2}and τ_{xy} = 4 kN/cm^2. Using the Mohrôs circle determine: a) the stresses acting on an element rotated through an angle θ = +40° (counter clockwise); b) the principal stresses; and c) the maximum shear stresses. **07**

OR

(b) Mention the limitation of Mohr's circle method and explain the state of stress for the three Mohr's circle shown below. **07**

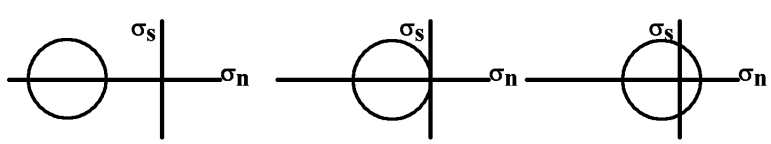

Fig. 1 Three cases depicting different state of stress

Q.3 (a) **07**

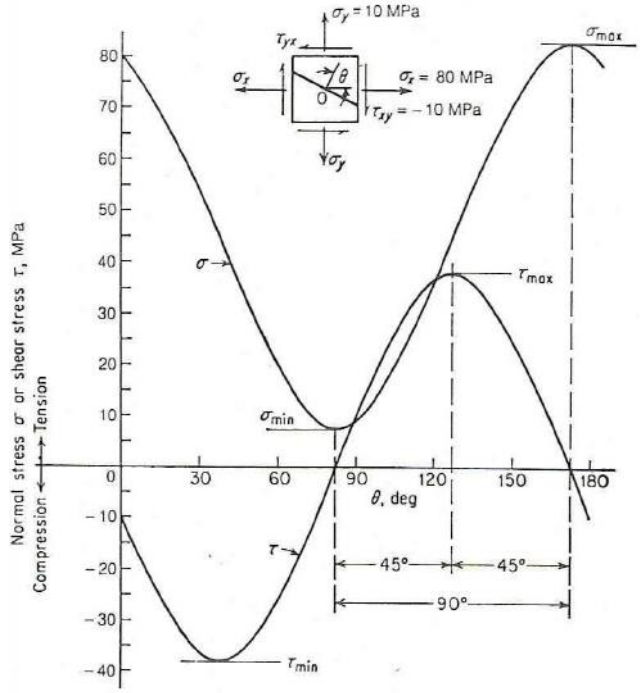

Fig. 2 Normal and shear stress variation in biaxial plane stress situation

The variation of normal and shear stress distribution for different planes passing from a point subtending angle ɗ, for the bi-axial state of stress is shown in the Fig. 2. Enlist four inferences as regards to maximum and minimum normal and shear stress and comment on the nature of the variation of stress for different planes passing from a point.

(b) The displacement field for a body is given by **07**

$U = (x^2+y)i + (3+z)j+(x^2+2y)k$

Write down the displacement gradient matrix at point (2,3,1)

OR

Q.3 (a) The displacement field for a body is given by **07**

$U = [(x^2+y^2+2)i + (3x+4y^2)j+(2x^3+4z)k]\ 10^{-4}$

What is the displaced position of a point originally located at point (1,2,3)?

(b) Two points P and Q in the undeformed body have coordinates (0,0,1) and (2,0,1) respectively. Assuming that the displacement field as **07**

$U = (x^2+y)i + (3+z)j+(x^2+2y)k$

Determine the distance between points P and Q after deformation.

Q.4 (a) "Incompressible materials do not show any volumetric strain, change in density and their Poisson's ratio is 0.5." Justify. Also show that second Lame's coefficient μ is same as modulus of rigidity G in usual notations. **07**

(b) A rubber cube is inserted in a cavity of the same form and size in a steel block and the top of cube is pressed by a steel block with a pressure of 'p' Pascals. Considering the steel to be absolutely hard and assuming that there is no friction between steel and rubber, find (i) the pressure of the rubber against the box walls if Poisson's ratio is 0.5 (ii) the extremum shear stress in rubber, if Poisson's ratio is Ò0.5. **07**

OR

Q.4 **(a)** A cubical element is subjected to following state of stress **07**

$$[\sigma_{ij}] = \begin{bmatrix} 100 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & -40 \end{bmatrix} \text{MPa}$$

Assuming the material to be homogeneous and isotropic, determine the principal shear strains and octahedral shear strain, if E = 2 x 10^5 MPa and Poisson's ratio as 0.25.

(b) Define bulk modulus, modulus of rigidity, Poisson's ratio, modulus of elasticity and explain its significance as regards to stress strain relationship in linear elastic solids. **07**

Q.5 **(a)** Derive the reciprocal theorem of Maxwell, Betti and Rayleigh showing that the forces of the first system acting through the corresponding displacements produced by any second force system do the same amount of work as that done by the second system of forces acting through the corresponding displacements produced by the first system of forces. **07**

(b) Derive Castigliano's first theorem and show that the partial differential coefficient of the strain energy function with respect to force F_r, gives displacement corresponding with F_r. **07**

OR

Q.5 **(a)** For a case of a thin hollow circular disk subjected to a temperature distribution which varies only with r and independent of ɑ and assuming that stress and displacement also do not vary over the thickness, show in usual notations that radial displacement **07**

$$u_r = (1+\nu)\alpha \frac{1}{r}\int_a^r Trdr + C_1 r + \frac{C_2}{r}$$

(b) Show how the strain matrix can be divided into hydrostatic stress and deviatoric stress. State the significance of deviatoric component of stress as regards to plastic deformation. **07**

****