Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – SUMMER • 2014

Subject code: 712901N **Date: 13-06-2014**

Subject Name: Mathematics for Researchers

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Obtain Fourier series for the function $f(x)=x^2$ in the interval $[-\pi,\pi]$. Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+....=\frac{\pi^2}{6}$. **07**

(b) Explain Rounding off and Inherent Errors. **07**

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Q.2 **(a)** State Trapezoidal Rule and using it evaluate the following integral. **07**

$\int_0^2 e^{x^2}\,dx$. (Take h=0.2)

(b) Detemine the largest eigen value and corresponding eigen vectors of the following matrix correct to three decimal places by using Power Method $\begin{bmatrix}4 & 1\\ 1 & 3\end{bmatrix}$. **07**

OR

(b) Round-off the number 37.46235 up to four significant figures and compute Absolute, Relative and Percentage errors. **07**

Q.3 **(a)** A curve is drawn to pass through the points given by the following table: **07**

x	1	1.5	2.0	2.5	3.0	3.5	4.0
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, x-axis and the lines x=1, x=4.

(b) The following data gives the velocity(m/sec) of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data. **07**

Time t	0	5	10	15	20
Velocity v	0	3	14	69	228

OR

Q.3 **(a)** Express the function $f(x)=x+|x|$ as a Fourier series in the interval $[-\pi,\pi]$. **07**

(b) Find the Fourier transform of the function f defined as follows: **07**

$$f(x)=\begin{cases}1-x^2; & |x|<1\\ 0 \quad ; & |x|>1\end{cases}$$

Q.4 **(a)** Solve the following system of equations by using Gauss elimination method. **07**

$x + y + z = 9;$

$2x - 3y + 4z = 13;$

$3x + 4y + 5z = 40.$

(b) State Cayley-Hamilton theorem and using it find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. **07**

OR

Q.4 **(a)** By using Gauss-Jordan method find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **07**

(b) Explain Escalator (Partition) Method. **07**

Q.5 **(a)** Using Taylorôs series method compute $y(0.2)$ correct to five decimal places, given that $\frac{dy}{dx} = 1 - 2xy$, y(0)=0. **07**

(b) Apply Runge-Kutta Method of order four to find an approximate value of y when x=0.2, given that $\frac{dy}{dx} = x^2 + y^2$, y(0)=1. **07**

OR

Q.5 **(a)** Find a real root of the equation $xe^x - 2 = 0$ correct to three decimal places by using Newton-Raphson Method. **07**

(b) Find a real root of the equation $x^3 - 3x - 5 = 0$ correct to three decimal places by using Bisection Method. **07**

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