Enrolment No._____

-GUJARAT TECHNOLOGICAL UNIVERSITY M. E. - SEMESTER – I • EXAMINATION – SUMMER • 2014

| M. E SEMESTER – I • EXAMINATION – SUMMER • 2014 | | | | | | |
|--|--------|---|-----|--|--|--|
| • | | 714101N Date: 13-06-2014 e: Mathematical Methods in Signal Processing | | | | |
| Subject Name: Mathematical Methods in Signal Processing Time: 02:30 pm - 05:00 pm Total Marks: 70 | | | | | | |
| Instructions: | | | | | | |
| | | npt all questions. | | | | |
| 2. 3. | | e suitable assumptions wherever necessary. res to the right indicate full marks. | | | | |
| 0. | i igui | es to the right indicate full marks. | | | | |
| Que 1) | a) | Compute transpose, inverse and rank of the matrixA, | [7] | | | |
| | | $\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | | | | |
| | | $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$ | | | | |
| | | $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ | | | | |
| | b) | Show that if A and B are open sets: i) $A \cup B$ is open ii) $A \cap B$ is open. | [7] | | | |
| Que 2) | a) | Obtain Z transform of $x(n) = (n)$ and $x(n) = a^n u(n)$ using basic definitions of Z transform. | [7] | | | |
| | b) | Explain Maximum Likely Hood Principle with suitable example. | [7] | | | |
| | , | OR | | | | |
| | b) | i) Let $f: X \to R$ be an arbitrary function defined on a set X. Show that | [4] | | | |
| | | d(x, y) = f(x) - f(y) is a Pseudometric. | | | | |
| | | ii) Show that for an induced norm $\ .\ $ over a real vector space | | | | |
| | | the parallelogram law is true. | [3] | | | |
| | | $ x + y ^{2} + x - y ^{2} = 2 x ^{2} + 2 y ^{2}.$ | | | | |
| Que 3) | a) | Obtain Fourier Transform of of $x(t) = (t)$ and $x(t) = (t-)$. | [7] | | | |
| | b) | Explain Bayes principle and show how to devise Bayes test for binary | [7] | | | |
| | | channel. OR | | | | |
| Que 3) | a) | i) Let x_t, y_t and v_t be discrete-time random processes with | [7] | | | |
| | | $y_t = x_t + v_t$ | | | | |
| | | $b(z^2)$ | | | | |
| | | And $S_{y}(z) = 1$ $S_{x}(z) = \frac{b(z^{2})}{a(z^{2})}$ | | | | |
| | | Where $b(z^2)$ and $a(z^2)$ are polynomials in z^2 with the degree of | | | | |
| | | $b(z^2)$ strictly lower than the degree of $a(z^2)$. Furthermore, assume $R_{xy}(t) = 0$. | | | | |
| | | Show that $H(z) = 1 - \frac{1}{S_v^+(z)}$ holds | | | | |
| | | Show that $H(2) = I = S_y^+(z)$ holds | | | | |
| | b) | Consider a zero-mean random vector $X = \{x_1, x_2, x_3\}$ with covariance | [7] | | | |
| | , | | | | | |
| | | $cov(X) = E[XX^T] = \begin{bmatrix} 7 & 4 & 2 \\ 7 & 4 & 2 \end{bmatrix}$ | | | | |
| | | $\operatorname{cov}(X) = E[XX^T] = \begin{bmatrix} 1 & .7 & .5 \\ .7 & 4 & .2 \\ .5 & .2 & 3 \end{bmatrix}$ | | | | |
| | | determine the optimal coefficients of the predictor of x_1 in terms of | | | | |
| | | x_2 and x_3 . $\ddot{x}_1 = c_2 x_2 + c_3 x_3$ | | | | |
| Oue 4) | a) | x_2 und x_3 . $x_1 - c_2 x_2 + c_3 x_3$ Explain contraction mapping theorem with its proof. | [7] | | | |

Que 4) a) Explain contraction mapping theorem with its proof.

[7]

| | b) | Explain LMS algorithm with application. OR | [7] |
|--------|----|--|-----|
| Que 4) | a) | Let $\ .\ $ is a norm satisfying the submultiplicative property and $A: x \to x$ is | [7] |
| | | an operator with. $ A < 1$. Then $(I - A)^{-1}$ exists, and $(I - A)^{-1} = \sum_{i=0}^{\infty} A^{i}$. | |
| | b) | i) Show that $(A^*)^{-1} = (A^{-1})^*$. ii) Show that if A has both a left inverse and a right inverse, they must be same. | [4] |
| Que 5) | a) | Discuss one method for phase estimation with block diagram. | [3] |
| 2000) | , | | [7] |
| | b) | Let $A: H \to H$ be a bounded linear operator on a Hilbertspace H. Show that: i) The adjoint operator A^* is linear. ii) The adjoint operator A^* is bounded iii) $ A = A^* $. | [7] |
| | | OR | |
| Que 5) | a) | ii) Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, using cayley Hamilton calculate A^5 . i) Determine the Jordan form of A. | [7] |
| | b) | $A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ | [4] |
| | | $\begin{bmatrix} 0 & 0 & 2 \end{bmatrix}$ ii) The eigenvalues of a self adjoin matrix are real. | [3] |
