Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER– II (Old course)• REMEDIAL EXAMINATION – SUMMER 2015

Subject Code: 1722309 **Date:16/05/2015**

Subject Name: Numerical Methods

Time: 02:30 pm to 5:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** What is an error? Explain various types of errors in numerical computation with proper examples. **07**

(b) Derive the formula for generating approximations of the root by using Newton-Raphson method. Use it to find a root of $x - 2\sin x = 0$ correct up to three significant figures taking $x_0 = \frac{\pi}{2}$. **07**

Q.2 **(a)** Write an algorithm for Bisection method. **07**
One real root of the equation $x - \cos x = 0$ lies between 0.5 and 1. Find the root correct up to two decimal places using bisection method.

(b) Water is flowing in trapezoidal channel at a rate of Q=20 m^3/s. The critical depth y for such a channel must satisfy the equation $0 = 1 - \frac{Q^2}{gA_c^2}B$ where g= 9.81m/s^2, A_c= the cross-sectional area (m^2), and B = the width of the channel at the surface(m). For this case, the width and the cross-sectional area can be related to depth y by B=3 + y and $A_c = 3y + \frac{y^2}{2}$.Solve for the critical depth using false position taking initial guesses of x_l=0.5 and x_u =2.5 up to three iterations and calculate the relative approximate error in the last iteration. **07**

OR

(b) Prove that Newton Raphson method is quadratic convergent method. **07**

Q.3 **(a)** Discuss the pitfalls of Gauss Elimination method. Also, discuss techniques for improving solutions. **07**

(b) Solve the following system of linear equations by using Gauss Siedel Method: **07**

$$\begin{aligned} 10x_1 + 2x_2 + x_3 &= 9 \\ 2x_1 + 20x_2 - 2x_3 &= -44 \\ -2x_1 + 3x_2 + 10x_3 &= 22 \end{aligned}$$

Take $x_1 = 0,\ x_2 = 0,\ x_3 = 0$ as an initial guess and $e_s = 5\%$

OR

Q.3 **(a)** The following system of equations was generated by applying the mesh current law to a circuit: **07**

$$\begin{aligned} 60I_1 - 40I_2 \qquad\qquad &= 200 \\ 40I_1 + 150I_2 - 100I_3 &= 0 \\ -100I_2 + 130I_3 &= 230 \end{aligned}$$

Solve for I_1, I_2, I_3 using matrix Inversion method.

(b) Explain and write an algorithm of Gauss Jordan Method. **07**

Q.4 **(a)** Describe the procedure to find the equation of a parabola $y = a_0 + a_1x + a_2x^2$ which best fits with the given points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ using least squares method. **07**

(b) The growth rate of bacteria(n) in a culture after t seconds is given below. **07**

t:	0	1	2	3	4	5
n:	32	45	65	93	125	185

Fit a curve of the form $n = ab^t$ and estimate n when t=2.5 seconds.

OR

Q.4 **(a)** Write an algorithm for Lagrangeôs Interpolation. **07**

Given the values

x:	5	7	11	13	17
f(x):	150	392	1452	2366	5202

Evaluate f(9) using Lagrangeôs interpolation formula.

(b) Use Newtonôs divided difference formula to the data given below to obtain a polynomial in x : **07**

x:	-4	-1	0	2	5
f(x):	1245	33	5	9	1335

Q.5 **(a)** Using geometrical interpretation derive formula for Trapezoidal Rule. **07**

The velocity v(km/min) of a vehicle which starts from rest, is given at fixed intervals of time t(min) as follows:

t:	2	4	6	8	10	12	14	16	18	20
v:	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes using Simpsonôs 1/3rd integration formula (when t=0, v=0).

(b) Write an algorithm for Eulerôs method. **07**

Using Runge-Kutta method of order 4, find y for x=0.1, given that $\frac{dy}{dx} = xy + y^2, \quad y(0) = 1$

OR

Q.5 **(a)** Using modified Eulerôs method, find an approximate value of y when x=0.4, given that $\frac{dy}{dx} = \log(x + y), \quad y(0) = 2$ taking step size h=0.2. **07**

(b) Use finite difference approach with $\Delta x = 2$ to solve the boundary value problem $7\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y + x = 0, \quad y(0) = 5$ and $y(10) = 8$. **07**

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