Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**ME - SEMESTER– I (New course)• REMEDIAL EXAMINATION – SUMMER 2015**

**Subject Code: 2710710** **Date:12/05/2015**

**Subject Name: Applied Linear Algebra**

**Time: 10:30 am to 1:00 pm** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Show that the set of vectors $\begin{bmatrix}1 & 3 & 3\end{bmatrix}^T$, $\begin{bmatrix}-1 & 1 & 2\end{bmatrix}^T$, $\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$ form a basis in $R^3$. With respect to this basis, find the coordinates of the following vectors : **07**

$(i)$ $\begin{bmatrix}2 & 2 & 3\end{bmatrix}^T$ $(ii)$ $\begin{bmatrix}-1 & 0 & -1\end{bmatrix}^T$

**(b)** Check whether the following sets of vectors are subspaces or not? **07**

(i) Set of all the vectors of the form $(a,b,c)$,where $b = a + c + 1$ of $R^3$.

(ii) Set of all the vectors of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$,where $a + b + c + d = 0$ of $M_{2\times 2}$.

(iii) Set of all the polynomials of the form $a_0 + a_1x$ ; where $a_0$ & $a_1$ both are integers of $P_1$.

(iv) Set of all the constant functions of $F(x)$.

**Q.2** **(a)** Find the Null Space, Row-Space and Column Space of the matrix : **07**

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

**(b)** Verify the Rank Nullity Therom for the following Transformations:

(i) $T : P_2 \to P_2$ ; $T\left(a_0 + a_1x + a_2x^2\right) = (a_1 - a_0)x + (a_2 - a_1)x^2$ **03**

(ii) $T : R^3 \to M_{22}$ ; $T(a,b,c) = \begin{bmatrix} a-b & b-c \\ a+b & b+c \end{bmatrix}$ **04**

**OR**

**(b)** Let $T: R^3 \to R^3$ be the linear operator defined by the formula **07**

$T(x_1, x_2, x_3) = (3x_1 + x_2, -2x_1 - 4x_2 + 3x_3, 5x_1 + 4x_2 - 2x_3)$.

Determine whether $T$ is one ï to ï one, if so, find $T^{-1}(x_1, x_2, x_3)$.

**Q.3** **(a)** For the matrix $A = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -3 & -1 \\ 4 & 1 & 3 \end{bmatrix}$, **07**

($i$) Write the quadratic form associated with it.

($ii$) Rewrite the matrix in the form of a symmetric matrix.

($iii$) Test if the quadratic form is positive definite.

**(b)** Verify Cayley ï Hamilton theorem for $A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & 3 \\ 1 & 4 & 3 \end{bmatrix}$ and also find $A^{-1}$. **07**

**OR**

**Q.3** **(a)** Find the Jordan canonical form of $A = \begin{bmatrix} -3 & 5 & 3 \\ -7 & 9 & 4 \\ 4 & -4 & 0 \end{bmatrix}$. **07**

**(b)** If $S_1 = \{(x, y, z) : x + 2y + z = 0\}$, $S_2 = \{(x, y, z) : x + y - z = 0\}$ are subspaces of $R^3$, then **07**

($i$) find a basis of $S_1 \cap S_2$,

($ii$) determine $\dim(S_1 + S_2)$,

and ($iii$) describe $S_1, S_2, S_1 \cap S_2$ and $S_1 + S_2$ geometrically.

**Q.4** **(a)** For vectors $x$ and $y$ in $C^n$, find ($i$) $\langle x, y \rangle$ ($ii$) $\langle y, x \rangle$, ($iii$) $\|x\|$, ($iv$) $\|y\|$, ($v$) normalized $x$, ($vi$) normalized $y$ and ($vii$) $d(x, y)$ where **07**

$x = \begin{bmatrix} 2 \\ i \\ 3 \end{bmatrix}$ and $y = \begin{bmatrix} 1+2i \\ 2i \\ 1 \end{bmatrix}$

**(b)** (i) Define the following terms : **03**

(a) Irreducible polynomial
(b) Annihilating polynomial
(c) Minimum polynomial

(ii) Find the minimum polynomial of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **04**

**OR**

**Q.4** **(a)** (i) Define the following terms : **03**

(a) Orthogonal matrix
(b) Proper and improper orthogonal matrix
(c) Unitary matrix

(ii) Show that if $A$ is orthogonal then $A^T$ and $A^{-1}$ are also orthogonal. And if $U$ is unitary then $U^T$ and $U^{-1}$ are also unitary. **04**

**(b)** Determine an orthonormal basis in $R^3$ from the set of independent vectors $x_1 = [1 \quad 0 \quad 1]^T$, $x_2 = [0 \quad 1 \quad 1]^T$, $x_3 = [1 \quad 1 \quad 0]^T$ using Gram ï Schmidt process. **07**

**Q.5** **(a)** Find the orthogonal projection of the vector $u = (-3, -3, 8, 9)$ on the subspace of $R^4$ spanned by the vectors $u_1 = (3, 1, 0, 1)$, $u_2 = (1, 2, 1, 1)$, $u_3 = (-1, 0, 2, -1)$ **07**

**(b)** Find the dominant eigen value and the corresponding eigen vector of $A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 5 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, by the power method. **07**

**OR**

**Q.5** **(a)** Two transformations $K$ and $L$ are given below. In each case, find the composite transformation $L \bullet K$ and $K \bullet L$ wherever it is defined : **07**

(i) $L(x, y, z) = (y, x+y-z)$, $\quad K(x, y) = (x-y, \ y, x)$

(ii) $L(x, y) = (x \cos\theta - y \sin\theta \ , \ x \sin\theta + y \cos\theta)$ ,
$K(x, y) = (x \cos\theta + y \sin\theta, -x \sin\theta + y \cos\theta)$

**(b)** Solve the following system of equations by Croutøs method : **07**

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

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