Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

ME- SEMESTER II– EXAMINATION – SUMMER 2015

Subject Code: 2720501 **Date: 26/05/2015**

Subject Name: Statistical Signal Analysis

Time: 2:30 PM – 5:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define a) sample space, b) event c) Random experiment, and explain with example. **07**
Find the sample space for the experiment of tossing a coin twice.

(b) A binary communications channel introduces a bit error in a transmission with probability p. Let X be the number of errors in n independent transmissions. Find the pmf of X. Find the probability of one or fewer errors. **07**

Q.2 **(a)** **(1.)** Define conditional probability and explain with example. **07**
(2.) State and explain conditions of events A and B to be statistically independent.

(b) Show that the poisson distribution can be used as a convenient approximation to the binomial distribution for large n and small p **07**

OR

(b) Let X be the number of bytes in a message, and suppose that X has a geometric distribution with parameter p. Find the mean of X. Also find Find the variance of the geometric random variable. **07**

Q.3 **(a)** **(1.)** Let $Y=X\cos(\omega t+\Theta)$ where Θ, ω are constants, and X is a uniform random random variable in the interval $(0, \pi)$, The random variable Y results from sampling the amplitude of the sinusoid with random amplitude X. Find the expected value of Y, i.e. $E(Y)$ and variance of Y. **04**

(2.) Let X be the number of heads in three tosses of a fair coin. Use the cdf to find the probability of the events A={1< XÒ 2}, B= {0.5ÒX < 2.5} and C= {1 ÒX < 2} . **03**

(b) **(1.)** Let $X(t)=\cos(\omega t+A)$, where A is uniformly distributed in the interval $(-\pi, \pi)$. Find the mean of $X(t)$ **03**

(2.) Define and explain (a) Gaussian Random Process (b) Strict sense Stationary Process. **04**

OR

Q.3 **(a)** Define and explain Mean Square Derivative of random process X(t) **07**

(b) Define and explain Ergodic process. Explain concept of ensemble average and time average. **07**

Q.4 **(a)** Suppose joint PMF of a bivariate random variable is given by **07**
$P_{X,Y}(x,y)$= 1/3 for (0,1);(1,0);(2,1)

(1) Are X and Y independent?
(2) Are X and Y uncorrelated?

(b) Explain Joint Moments, Correlation, and Covariance. Also prove that covariance of independent random variable is zero. **07**

OR

Q.4	**(a)**	Find the normalization constant c and the marginal pdfôs for the following joint pdf $$f_{X,Y}(x, y) = \begin{cases} ce^{-x}e^{-y} & 0 \le y \le x < \infty \\ 0 & \text{elsewhere.} \end{cases}$$ Find P[X + Y < 1].	**07**
	(b)	**(1.)** Find E[E[Y|X]]. Where E[.] is expected operator	**04**
		(2.) Let Z=X+Y. Prove that E[Z]=E[X]+E[Y].	**03**
Q.5	**(a)**	Consider an experiment of drawing randomly three balls from an urn containing two red, three white and four blue balls. Let *(X, Y)* be a random variable, where *X* and *Y* denotes respectively the number of red and white balls chosen, Find range of (*X,Y*), joint pmfôs of (*X,Y*), marginal pmfôs of *X* and *Y*. Are *X* and *Y* independent?	**07**
	(b)	Explain Central limit Theorem with proof.	**07**

OR

Q.5	**(a)**	**(1.)** Let $X(t)=a*\cos(2\pi f_0 t+B)$, where *B* is the uniformly distributed in the interval $(0, 2\pi)$. Find Power Spectral Density (PSD).	**04**
		(2.) Derive necessary Explain Markov and Chebshev inequalities	**04**
	(b)	Give answer of following questions. (i) Let $Z = X + Y$. Find $F_Z(z)$ and $f_Z(z)$ in terms of the joint pdf of X and *Y*. (ii) Let *X* and *Y* be jointly Gaussian random variables. Derive the joint characteristic function of *X* and *Y* using conditional expectation.	**07**

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