Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME- SEMESTER II– EXAMINATION – SUMMER 2015

Subject Code: 2720714 **Date: 26/05/2015**

Subject Name: Modern Control System

Time: 2:30 PM – 5:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) What are the major advantages of state space representation over transfer function representation . Hence define state, state variables & state space **4+3**

(b) Discuss the following nonlinearities 1. Saturation 2. Relay 3. Dead Zone **07**

Q.2 (a) The transfer function of a system is $G(s) = 2s^2 + 6s + 7/(s^3 + 4s^2 + 5s + 2)$ Obtain the state space representation in diagonal canonical form. **07**

(b) Prove that the eigenvalues are invariant under linear transformation. Find the eigenvalues & eigenvectors for matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ **07**

OR

(b) Consider the state model $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ where u is unit step input. Compute the state transition matrix & therefrom find the state response given $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ **3+4**

Q.3 (a) Define óControllabilityô. Hence, prove the Gilbertôs method for testing the controllability of a given system. **07**

(b) Consider system given by $\dot{X} = AX + Bu$ where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ Design a full state feedback controller using Ackermannôs formula to place the closed loop poles at s = $-2 + j3$ & $s = -6$. **07**

OR

Q.3 (a) Check the controllability & observability for a state space system represented with A = $\begin{bmatrix} 3 & -1 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $C^T = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ where A,B,C have usual meaning **07**

(b) What is an óObserverô. Obtain the state space representation for a controller-observer based system & show that the pole placement design & observer design are independent of each other. **07**

Q.4 (a) Define Positive Definiteness and Positive Semi Definiteness of a quadratic function. Hence, check the definiteness of the following function **07**

1. $Q_1 = -x^2 + (3x_1 + 2x_2)^2$
2. $Q_2 = 2x_1^2 - 2x_1x_2 + 2x_2^2 - 2x_2x_3 + +4x_3^2$

(b) Obtain th control input that minimizes the performance index $J = \int_0^\infty (x_1^2 + u^2)$ **07**

for the system $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$

OR

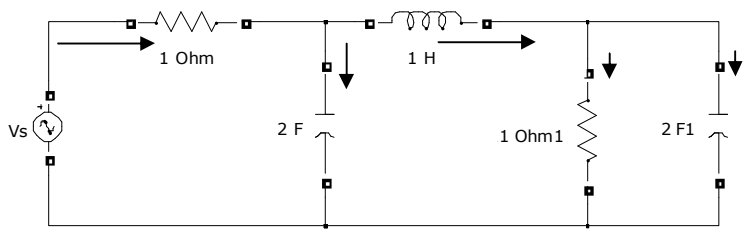
Q.4 **(a)** Explain how the Phase plane method is useful in determining the behavior of nonlinear systems. **07**

(b) Explain Liapunovôs Stability Criteria and therefrom state Liapunovôs Theorem. Determine the stability of the system given by **04+03**

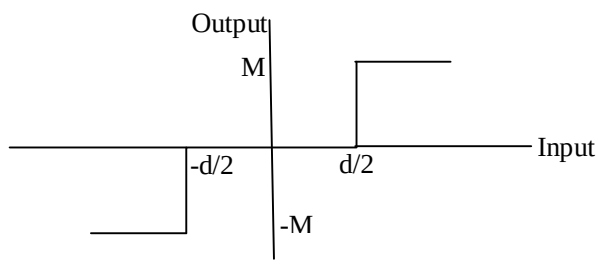
$\dot{x}_1 = 2x_2 - x_1(x_1^2 + x_2^2)$ $\qquad$ $\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2)$

Assume a suitable Liapunov function.

Q.5 **(a)** Obtain the state space representation for the system shown below **07**



(b) Obtain the describing function for the following nonlinearity i.e a Relay with dead zone) **07**



OR

Q.5 **(a)** Explain calculus of variation to find maxima or minima. With the help of a suitable performance index, find the equation of the curve between points (x_0 ,t_0) and (x_1 ,t_1) such that the arclength is minimum. **3+4**

(b) Consider an autonomous system represented by $\dot{x} = \mathbf{F}(x)$.For a second order system, explain the nature of the phase plane trajectories when i. eigenvalues are real, distinct & negative ii. eigenvalues are complex conjugates with negative real parts iii. eigenvalues are imaginary & hence discuss the stability of its singular point. **07**

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