Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER– I (OLD course)• EXAMINATION – SUMMER 2015

Subject Code: 710401 **Date:11/05/2015**

Subject Name: **Statistical Signal Analysis**

Time: 10:30 am to 1:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Prove the following statements **07**

1. For any event A, P(A) = 1 ï P(Aô)
2. For any events A & B, P(A+B) = P(A) + P(B) ï P(AB)

(2) A committee of 5 persons is to be selected randomly from a group of 5 men and 10 women.

(a) Find the probability that the committee consists of 2 men and 3 women.

(b) Find the probability that the committee consists of all women.

(b) Explain in detail Central Limit Theorem. Given x = cosď and y = sinď, where ď is an RV uniformly distributed in the range (0, 2π), show that x and y are uncorrelated but are not independent. **07**

Q.2 **(a)** Explain in detail various definitions of probability and their roles in our investigations. **07**

Two manufacturing plants produce similar parts. Plant 1 produces 1,000 parts, 100 of which are defective. Plant 2 produces 2,000 parts, 150 of which are defective. **A** part is selected at random and found to be defective. What is the probability that it came from plant 1?

(b) **A** lot of 100 semiconductor chips contain 20 that are defective. Two chips are selected at random, without replacement, from the lot. **07**

(a) What is the probability that the first one selected is defective?

(b) What is the probability that the second one selected is defective given that the first one was defective?

(c) What is the probability that both are defective?

OR

(b) A player chooses any 8 numbers out of 49 numbers. Eight balls are drawn randomly without replacement from 49 balls numbered 1 through 49. **07**

(a) Find the probability of matching all 8 balls to the 8 numbers chosen by the player

(b) Find the probability of matching exactly 5 balls

(c) Find the probability of matching exactly 6 balls

Q.3 **(a)** Find f(x) if F(x) = (1- $e^{-\alpha x}$) $U(x- c)$ and plot the Distribution function and Density function. **07**

(b) The PDF of amplitude x of a certain signal x(t) is given by **07**

$$p_x = 0.5\,|x|\,e^{-|x|}$$

1. Find the probability that $x \geq 1$
2. Find the probability that $-1 < x \leq 2$
3. Find the probability that $x \leq -2$.

OR

Q.3 **(a)** Determine mean, mean square and the variance of the random variable X for the following $\mathbf{P_x(X) = 0.5\,|x|\,e^{-|x|}}$ **07**

(b) A binary source generates digits 1 and 0 randomly with equal probability. Assign probabilities to the following events. In ten digits generated by the source (a) there are exactly three 1ôs and seven 0ôs (b) there are at least six 0ôs. (c) there are at least five 1ôs **07**

Q.4 (a) Consider an experiment of tossing two coins three times. Coin A is fair, but coin B is not fair, with P(H) = 1 / 4 and P(T) = 3 / 4. Consider a bivariate RV (X, Y), where X denotes the number of heads resulting from coin A and Y denotes the number of heads resulting from coin B. **07**

(i) Find the range of (X, Y).
(ii) Find the joint pmf'ôs of (X, Y).
(iii) Find P(X = Y), P(X > Y), and P(X + Y ≤ 4).

(b) Mr. shah receives at least 09 calls a day. Let **A** be the event he receives at most 15 calls a day; Let **B** be the event he receives at least 12 calls a day; Let **C** be the event he receives between 15 and 20 calls a day. Assume: P(**A**) = 0.60, P(**B**) = 0.85, P(**C**) = 0.25, P(**AUC**) = 0.40 . Find the probability of the event **E** that **07**

(a). he receives at least 16 calls a day.
(b). he receives 10 or 11 calls a day (inclusive).
(c). he receives between 12 and 15 calls a day (inclusive).
(d). he receives exactly 15 calls a day.
(e). he receives at least 16 calls a day or less than 12 calls a day.

OR

Q.4 (a) Consider an experiment of drawing randomly three balls from an urn containing two red, three white, and four blue balls. Let (X, Y) be a bivariate RV where X and Y denote, respectively, the number of red and white balls chosen. **07**

(i) Find the range of (X, Y).
(ii) Find the joint pmfôs of (X, Y).
(iii) Find the marginaI pmf'ôs of X and Y.
(iv) Are X and Y independent?

Q.4 (b) Show that the Poisson distribution can be used as a convenient approximation to the binomial distribution for large n and small p. **07**

Q.5 (a) What do you mean by random process? Classify random process and explain two them in detail. Show that random process **07**

x(t) = A cos (ω_c t + d) , where d is an RV uniformly distributed in the range (0, 2π), is a wide ï sense stationary process.

(b) Explain in detail CDF, PDF and joint distribution of RVs with examples. **07**

OR

Q.5 (a) Explain in detail with examples following terms related to the random process **07**

1.ensemble
2. sample function
3. ergodic processes

(b) Production line manufactures 1000-ohm (R) resistors that have 10 percent tolerance. Let X denotes the resistance of a resistor. Assuming that X is a normal RV with mean 1000 and variance 2500, find the probability that a resistor picked at random will be rejected. **07**

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