Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER– I (OLD course)• EXAMINATION – SUMMER 2015

Subject Code: 710901 **Date:11/05/2015**

Subject Name: THEORY OF ELASTICITY

Time: 10:30 am to 1:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Show that if stress vectors on three mutually perpendicular plane passing through the point are known then the stress vector on any other arbitrary plane at that point can be determined. **07**

(b) The state of stress varies from point to point in a body, however, if the body is in equilibrium then show that all the points throughout the volume obeys governing equations of equilibrium. **07**

Q.2 **(a)** Prove that six conditions of compatibility, imposed on the components of strain, are necessary and sufficient to insure a continuous single-valued displacement field. **07**

(b) The cross-section of the wall of a dam is shown in Fig.1. The pressure of water on face OB is also shown. With the axes Ox and Oy, as shown in Fig.1, the stresses at any point (x, y) are given by (ɔ= specific weight of water and ɟ = specific weight of dam material) **07**

$$\sigma_x = -\gamma y$$

$$\sigma_y = \left(\frac{\rho}{\tan\beta} - \frac{2\gamma}{\tan^3\beta}\right)x + \left(\frac{\gamma}{\tan^2\beta} - \rho\right)y$$

$$\tau_{xy} = \tau_{yx} = -\frac{\gamma}{\tan^2\beta}x$$

$$\tau_{yz} = \tau_{zx} = \sigma_z = 0$$

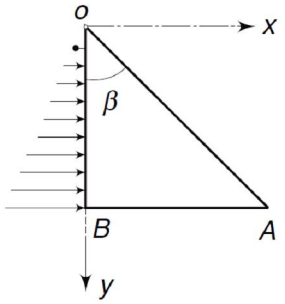

Fig.1

OR

(b) The following displacement field is imposed on a body **07**

$u = (xy\,i + 3x^2z\,j + 4\,k)10^{-2}$

Consider a point P and a neighboring point Q where PQ has the following direction cosines

$n_x = 0.200$, $n_y = 0.800$, $n_z = 0.555$

Point P has coordinates (2, 1, 3). If PQ = qps, find the components of P'Q' after deformation.

Q.3 **(a)** Show in usual notations that the principal axes of strain remain orthogonal after strain. **07**

(b) Differentiate homogeneity and isotropy as regards to the material property and for the isotropic material correlate principal stress and principal strain through Generalized Hooke's Law. **07**

OR

Q.3 **(a)** The displacement field in micro units for a body is given by **07**
$\mathbf{u} = (x^2 + y)\mathbf{i} + (3 + z)\,\mathbf{j} + (x^2 + 2y)\mathbf{k}$
Determine the principal strains at (3, 1, ï 2) and the direction of the minimum principal strain.

(b) A rubber cube as shown in Fig. 2 is inserted in a cavity of the same form and size in a steel block and the top of the cube is pressed by a steel block with a pressure of p pascals. Considering the steel to be absolutely hard and assuming that there is no friction between steel and rubber, find (i) the pressure of rubber against the box walls, and (ii) the extremum shear stresses in rubber. **07**

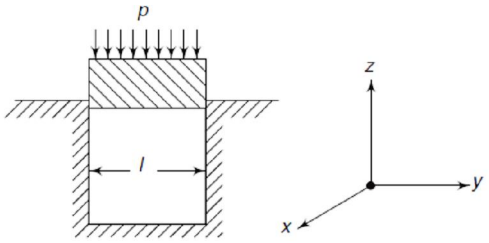

Fig.2

Q.4 **(a)** The cantilever beam shown in Fig. 3 is subjected to a bending moment $M = F_1$ at point 1, and in Fig.3, it is subjected to a concentrated load $P = F_2$ at point 2. The point 2 is 2/3 L from the fixed end. Verify the reciprocal theorem. **07**

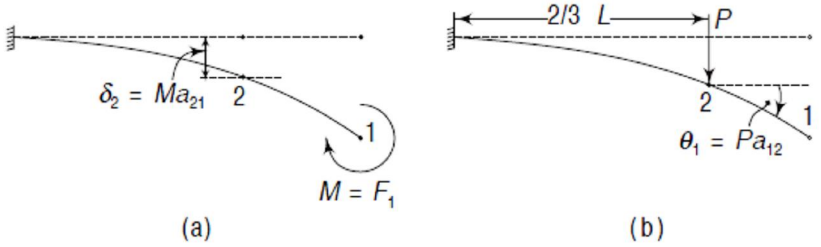

Fig.3

(b) For the cantilever of total length L shown in Fig.4, determine the deflection at end A. Neglect shear energy. **07**

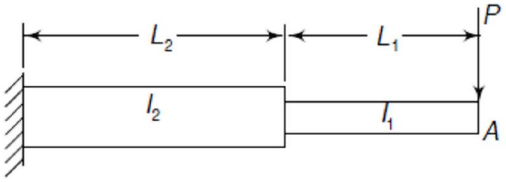

Fig.4

OR

Q.4 **(a)** State Castlingo's first theorem and show through an engineering example that this theorem is useful in determining the displacement of the structures as well as solution of many statically indeterminate structure. **07**

(b) Show that when an elastic body is subjected to several forces, one cannot obtain the total elastic energy by adding energies caused by individual forces. **07**

Q.5 **(a)** Let the inner surface of a hollow sphere be at temperature Ti and the outer surface at temperature zero. Let the system be in a steady heat flow condition. The temperature distribution is then given by **07**

$$T = \left(\frac{T_i a}{b-a}\right)\left(\frac{b}{r} - 1\right)$$

Determine the stress distribution, if 'a' is the inner radius and 'b' is the outer radius of the sphere.

(b) The inner surface of a hollow tube is at temperature Ti and the outer surface at zero temperature. Assuming steady-state conditions, calculate the stresses. What are the values of σ_θ and σ_z near the inner and outer surfaces? **07**

OR

Q.5 **(a)** Consider a thin disk subjected to a temperature distribution which varies only with *r* and is independent of θ. Determine the values of σ_θ and σ_r if the disk is solid with radius 'b'. **07**

(b) Consider a thin disk subjected to a temperature distribution which varies only with *r* and is independent of θ. Determine the values of σ_θ and σ_r if the disk is hollow with inner radius 'a' and outer radius 'b'. **07**

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