Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER– III • EXAMINATION – SUMMER 2015

Subject Code: 730403 **Date: 02/05/2015**

Subject Name: Optimization Techniques

Time: 2:30 pm to 5:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **.**

Q.1 **(a)** (i) Write a MATLAB program to find the maxima or minima of the function using Symbolic Mathematics Toolbox **04**

$$f(x) = 10x^6 - 48x^5 + 15x^4 + 200x^3 - 120x^2 - 480x + 100.$$

(ii) Find the critical point of the function **03**

$$f(x_1, x_2) = -x_1^2 - x_2^2 + 2x_1x_2 - x_3^2 + 6x_1x_3 + 4x_1 - 5x_3 + 2.$$

(b) An electric light is placed directly over the center of a circular plot of lawn 100 m in diameter. Assuming that the intensity of light varies directly as the sine of the angle at which it strikes an illuminated surface, and inversely as the square of its distance from the surface, how high should the light be hung in order that the intensity may be as great as possible at the circumference of the plot? **07**

Q.2 **(a)** Check whether the function **07**

$$f(x_1, x_2, x_3) = 4x_1^2 + 3x_2^2 + 5x_3^2 + 6x_1x_2 + x_1x_3 - 3x_1 - 2x_2 + 15$$

is convex. Also, express the function in matrix form as

$$f(X) = \frac{1}{2}X^T[A]X + B^TX + C$$

and determine whether the matrix [A] is positive definite, negative definite, or indefinite.

(b) Two types of printed circuit boards A and B are produced in a computer manufacturing company. The component placement time, soldering time, and inspection time required in producing each unit of A and B are given below: **07**

Circuit Board	*Time required per Unit (min) for*		
	Component Placement	Soldering	Inspection
A	16	10	4
B	10	12	8

If the amounts of time available for component placement, soldering and inspection are 1500, 1000, and 500 person-minutes respectively, determine the number of units of A and B to be produced for maximizing the production.

OR

(b) Minimize $f = 20x_1 + 16x_2$ subject to **07**

$$x_1 \geq 2.5, \quad x_2 \geq 6, \quad 2x_1 + x_2 \geq 17, \quad x_1 + x_2 \geq 12$$
$$x_1 \geq 0, \quad x_2 \geq 0$$

using dual simplex method.

(a) Use Dichotomous Search Method to find the minimum of **07**

$$f(\lambda) = 3\lambda^5 - 10\lambda^3 - 45\lambda + 1$$

in the interval $(1.00, 3.00)$ to within 10% of the exact value. Take $\delta = 0.001$.

(b) Use Quasi-Newton's Method to minimize the function **07**

$$f(\lambda) = 0.65 - \frac{0.75}{1+\lambda^2} - 0.65\,\lambda\, tan^{-1}\frac{1}{\lambda}$$

with the starting point $\lambda_1 = 0.1$ and the step size $\Delta\lambda = 0.01$. Use $\varepsilon = 0.01$ for checking the convergence.

OR

Q.3 **(a)** Use Golden Section Method with $n = 6$ to maximize the function **07**

$$f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda$$

in the interval $[0, 3]$.

(b) Minimize $f(X) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$ with the starting point $X_1 = [-1\ 1]^T$ using Newton's method. Perform only two iterations. **07**

Q.4 **(a)** Write the algorithm of unrestricted search with fixed step size. Explain the need to modify the algorithm which use accelerated step size. **07**

(b) Write the algorithm of Fletcher-Reeves Method. Minimize the function **07**

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

starting from the point $X_1 = [0\ 0]^T$.

OR

Q.4 **(a)** Use Fibonacci Method to minimize the function **07**

$$f(\lambda) = \frac{\lambda}{log\lambda}$$

with $n = 6$ in the interval $(0, 3)$.

(b) Write some applications of optimization in engineering. **07**

Q.5 **(a)** Use Cauchy's steepest descent method to minimize **07**

$$f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$$

from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform only two iterations.

(b) Write the algorithm of Sequential Linear Programming for minimization of a constrained multi-variable non-linear optimization problem. **07**

OR

Q.5 **(a)** Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ from the starting point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ using Univariate Method. Take $\varepsilon = 0.01$. Perform two iterations. **07**

(b) Write the algorithm of Exterior Penalty function Method. **07**

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