Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E –Ist SEMESTER–EXAMINATION – JULY- 2012

Subject code: 710107N **Date: 07/07/2012**

Subject Name: Quantum Theory & Algorithm Design

Time: 2:30 pm – 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Suppose **07**

$$|\Psi\rangle = 3|0\rangle - 2i\,|1\rangle,\ |\emptyset\rangle = |0\rangle + 5\,|1\rangle$$

(i) Show that these states obey the Cauchy-Schwarz and triangle inequalities.

(i) Normalize the states.

(b) Write the algorithm for LUP- DECOMPOSITION and solve the equation. **07**

$$\begin{pmatrix} 1 & 5 & 4 \\ 2 & 0 & 3 \\ 5 & 8 & 2 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} 12 \\ 9 \\ 5 \end{pmatrix}$$

Using LUP decomposition.

Q.2 **(a)**

(i) Show that the set **04**

$$|a\rangle = \begin{pmatrix} 1 \\ 2 \end{pmatrix},\ |b\rangle = \begin{pmatrix} -1 \\ 1 \end{pmatrix},\ |c\rangle = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

Is linearly dependent; that is, one of the vectors can be written as a linear combination of the other two.

(ii) Explain cardinality of the set. The set of natural numbers divisible by 7 and 9 are countable or uncountable? **03**

(b)

(i)Explain positive definite matrices and show that for any matrix A with full column ranks the matrix $A^T A$ is positive definite. **04**

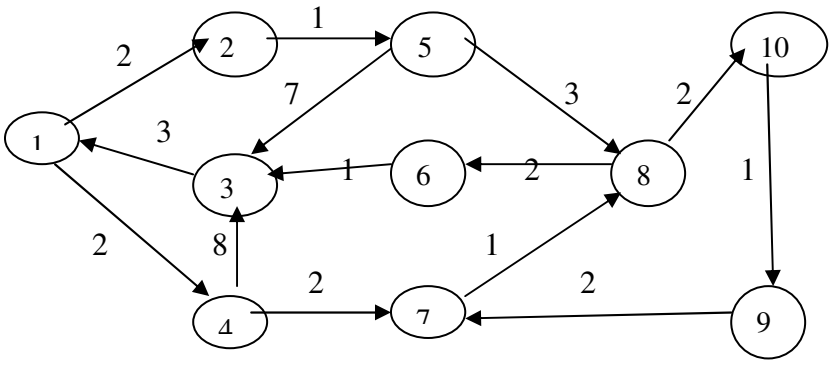
(ii)Explain **03**

a) ORTHONORMAL BASIS SET.

b) Important properties of trace.

OR

(b) List the shortest path from vertex 1 to each vertex of figure. Use greedy method. **07**



Q.3 **(a)** Suppose that a system is in the state **07**

$$|\psi\rangle = \sqrt{\frac{2}{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$$

(i) Compute Tr (ρ) and Tr (ρ^2). Is this a mixed state ?
(ii) Find <X> for this state.

(b) Find $X \otimes Y|\psi\rangle$ where **07**

$$|\psi\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

OR

Q.3 **(a)** Describe the action of operators $P_0 \otimes I$ and $I \otimes P_1$ on the state **07**

$$|\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.$$

(b) A three state system is in the state **07**

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|2\rangle$$

Write down necessary projection operators and calculate the probabilities Pr(0), Pr(1) and Pr(2)

Q.4 **(a)** For the density matrix given by **07**

$$\rho = \frac{1}{5}\begin{pmatrix} 3 & 1-i \\ 1+i & 2 \end{pmatrix}$$

(A) Is this a mixed state?
(B) Find<X>,<Y> and <Z> for this state.

(b) Use the divide and conquer strategy on merge sort to illustrate the operation on the array **07**
A=(3,41,52,26,38,57,9,49)
And write the algorithm.

OR

Q.4 **(a)** A System is in the state **07**

$$|\psi\rangle = \frac{1}{\sqrt{8}}|00\rangle + \sqrt{\frac{3}{8}}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

(a) What is the probability that measurement finds the system in state |Φ>=|01>?

(b) What is the probability that measurement finds the first qubit in the state |0>? What is the state of system after measurement?

(b) What is the difference between recursive and iterative solution of 0/1 knapsack problem using dynamic programming **07**

Q.5 **(a)**

(i) How the problem of single source shortest paths with negative casts will be solved? **04**

(ii) Determine a suitable value for the hash functions division D when linear probing is used. **03**

a) N=50, $S_n \leq 3$, $U_n \leq 20$

b) N =500, $S_n \leq 5$, $U_n \leq 60$

(b)

(i) Draw the Huffman tree for the weights Frequencies [3,7,9,12,15,20,25] **04**

(ii) How will an element be searched, an element be inserted and deleted from binary search trees. **03**

OR

Q.5 **(a)**

(i) write the breath first search algorithm for graph searching **04**

03

(ii) Let T be an ordered tree with more than one node. Is it possible that the preorder traversal of T visiting the nodes in the same order as the post order traversal of T? If so, give an example; otherwise argue why this cannot occur. Likewise, is it possible that the preorder traversal of T visits the nodes in the postorder traversal of T? If so, give an example; otherwise argue why this cannot occur.

(b)

(i) Write the algorithm for Euler tour traversal of a binary tree. **04**

(ii) When a binary relation is antisymmetric? Give example of a relation that is reflexive, transitive but antisymmetric. **03**

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