Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E –Ist SEMESTER–EXAMINATION – JULY- 2012

Subject code: 710904N Date: 13/07/2012

Subject Name: Optimization Techniques

Time: 2:30 pm – 05:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Classify engineering optimization techniques with suitable examples based on 07

a. Nature of constraints
b. Nature of equations involved
c. Deterministic nature of variables

(b) Explain the importance of integer programming with examples. 07

Q.2 (a) A company is manufacturing two different types of products, A and B. Each product has to be processed on two machines M_1 and M_2. Product A requires 2 hours on machine M_1 and 1 hour on machine M_2. Product B requires 1 hour on machine M_1 and 2 hours on machine M_2. The available capacity of machine M_1 is 104 hours and that of machine M_2 is 76 hours. Profit per unit of product A is Rs. 6 and that for B is Rs. 11. 07

i) Formulate the problem
ii) Find out the optimal solution by simplex method

(b) Figure 1 shows two frictionless rigid bodies A and B connected by three linear elastic springs having spring constants k_1, k_2, and k_3. The springs are at their natural positions when the applied force P is zero. Find the displacements $x_1$and x_2 under the force P by using the principle of minimum potential energy. 07

OR

(b) In a two-stage compressor, the working gas leaving the first stage of compression is cooled (by passing it through a heat exchanger) before it enters the second stage of compression to increase the efficiency. The total work input to a compressor (W) for an ideal gas, for isentropic compression, is given by, 07

$$W = C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} + \left(\frac{P_3}{P_2} \right)^{(k-1/k)} - 2 \right]$$

Where, C_p is the specific heat of the gas at constant pressure, k is the ratio of specific heat at constant pressure to that at constant volume of the gas, and T_1 is the temperature at which the gas enters the compressor. Find the pressure, P_2, at which inter cooling should be done to minimize the work input to the compressor. Also determine the minimum work done on the compressor.

Q.3 (a) A beam of uniform rectangular cross section is to be cut from a log having a circular cross section of diameter 2a. The beam has to be used as a cantilever beam (the length is fixed) to carry a concentrated load at the free end. Find the dimensions of the beam that correspond to the maximum tensile (bending) stress carrying 07

capacity.

(b) Solve following problem using Lagrange multiplier method. 07

Maximize $5 - (x_1-2)^2 - 2(x_2-1)^2$

Subject to $x_1 + 4x_2 = 3$

OR

Q.3 (a) Explain the solution by the Method of Constrained Variation. 07
Derive the equation:

$$\left(\frac{\partial f}{\partial x_1}\frac{\partial g}{\partial x_2} - \frac{\partial f}{\partial x_2}\frac{\partial g}{\partial x_1}\right)_{(x_1^*, x_2^*)} = 0$$

Where, (x_1^*, x_2^*) is an extreme point.

Assume the problem as:
Minimize $f(x_1,x_2)$
Subjected to $g(x_1,x_2)=0$

(b) Use Big M method to solve the following LPP 07

Minimize, $z = 12x_1 + 20x_2$

Subject to, $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$

$x_1, x_2 \geq 0$

Q.4 (a) Use Kuhn-Tucker method to solve following example. 07

Minimize $f = 10x_1 - x_1^2 + 10x_2 - x_2^2$

Subject to

$14 >= x_1 + x_2$

$-x_1 + x_2 <= 6$ and $x_1, x_2 >= 0$

(b) What is duality in LPP? Write the dual of the following LPP: 07

$Z = 4x_1 + 6x_2 + 18x_3$

Subject to, $x_1 + 3x_2 \geq 3$

$x_2 + 2x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$

OR

Q.4 (a) Use two phase simplex method to, 07

Minimize $Z = 5x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1$

$x_1 + 4x_2 \geq 6$

$x_1, x_2 \geq 0$

(b) Solve problem using Lagrange Multiplier method. 07

Maximize $x^3 - 3x$

Subject to

$x <= 2$

Q.5 (a) Solve the following LP problem using branch and bound method: 07

Maximize $3x_1+4x_2$

Subjected to:

$7x_1 + 11x_2 \leq 88$

$3x_1 - x_2 \leq 12,$

$x_1 \geq 0$

$x_2 \geq 0,$

x_1, x_2 are integer

(b) Explain dynamic programming. How is it different from linear programming? 07

Write four steps of dynamic programming.

OR

Q.5 (a) Find the optimum solution of the following LPP: 07

Max $Z=X_1+X_2$

Subjected to

$3X_1+2X_2 \leq 5,$

$X_2 \leq 2,$

$X_1 + X_2 \leq 0$, and X_1 is an integer. Use cutting plane method.

(b) A corporation has 5 lacs to allocate to its three plants for possible expansion. Each plant has submitted a number of proposals on how it intends to spend the money. Each proposal gives the cost of the expansion (c) and the total revenue expected (r). The following table gives the proposals generated: 07

Proposal	Plant 1.		Plant 2.		Plant 3	
	c_1	r_1	c_2	r_2	c_3	r_3
1	0	0	0	0	0	0
2	1	5	2	8	1	4
3	2	6	3	9	—	—
4	—	—	4	12	—	—

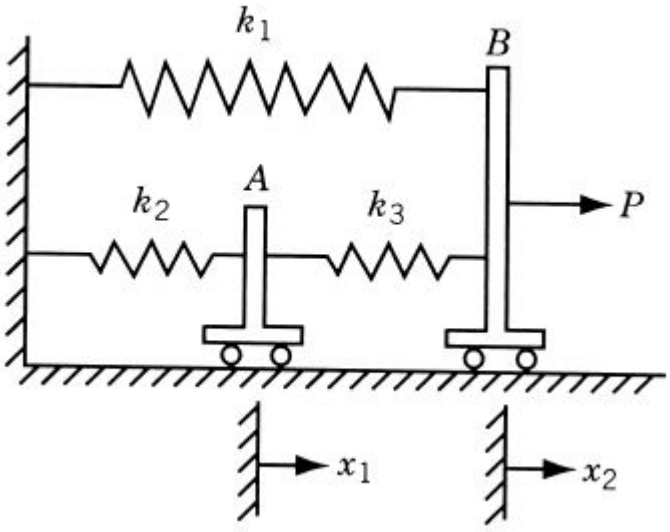

Figure 1

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