Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER-1 (NEW) EXAMINATION – WINTER 2016

Subject Code: 2710710 **Date:03/01/2017**

Subject Name: Applied Linear Algebra

Time: 2:30 pm to 5:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) In a 3 - space R^3, let W be the set of all vectors (x_1, x_2, x_3) that satisfy the equation $x_1 - x_2 - x_3 = 0$ .Find a basis for the subspace W . **03**

(ii) Consider $U = \{(x, y) \in R^2 : x - y = 0\}$ and $V = \{(x, y) \in R^2 : 3x + y = 0\}$. Show that they are subspaces of R^2 .Does R^2 is a direct sum of U and V ? **04**

(b) State rank-nullity theorem. **07**

Find the rank and the nullity of

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & 2 \\ 1 & 2 & -2 & -4 & 3 \end{bmatrix}$$

Q.2 **(a)** Let $V = \{(x, y, z, u) \in R^4 : y + z + u = 0\}$, **07**

$W = \{(x, y, z, u) \in R^4 : x + y = 0, \ z = 2u\}$

be two subspaces of R^4. Find bases for V , W ,$V + W$ and $V \cap W$.

(b) Let A and B be subspaces of a finite dimensional vector space V . Prove that $A + B$ and $A \cap B$ are finite dimensional and **07**

$\dim A + \dim B = \dim (A + B) + \dim (A \cap B)$

OR

(b) Find the rank of A as a function of x : **07**

$$A = \begin{bmatrix} 2 & 2 & -6 & 8 \\ 3 & 3 & -9 & 8 \\ 1 & 1 & x & 4 \end{bmatrix}$$

Q.3 **(a)** (i) Find the matrix of reflection about the line $y = x$ in R^2. **03**

(ii) Is there a linear transformation $T : R^3 \to R^2$ such that $T(1,1,1) = (1,0)$, $T(1,1,0) = (2,-1)$ and $T(1,0,0) = (4,3)$? If yes, then find an expression for $T(x)$. **04**

(b) **07**

For the matrix $A = \begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$,

(i) diagonalize the matrix A

(ii) find the eigen values of $A^{10} + A^7 + 5A$

OR

Q.3 **(a)** **03**

(i) Show that $A = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ is an idempotent matrix.

(ii) Let $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $f(x) = 2x^3 - 4x + 5$ **04**

1. Find A^2, A^3, $f(A) = 2A^3 - 4A + 5I$
2. Show that A is a zero of the polynomial $g(x) = x^2 + 2x - 11$

(b) Find the maximum and minimum values of the quadratic form **07**
$x_1^2 + x_2^2 + 2x_3^2 - 2x_1x_2 + 4x_1x_3 + 4x_2x_3$ subject to the constraint $x_1^2 + x_2^2 + x_3^2 = 1$

Q.4 **(a)** Compute the minimal polynomial of **07**

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

(b) **07**

For the matrix $A = \begin{bmatrix} 2i & 0 & 0 \\ i & -1 & -i \\ -1 & 0 & 2i \end{bmatrix}$, find a unitary matrix U such that $U^{-1}AU$ is an upper triangular matrix.

OR

Q.4 **(a)** Find an orthonormal basis for the Euclidean space R^3 by applying the Gram – Schmidt orthogonalization to the three vectors **07**
$x_1 = (1,0,0)$, $x_2 = (3,7,-2)$, $x_3 = (0,4,1)$.

(b) Find the Jordan canonical form of the matrix **07**

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 4 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

Q.5 **(a)** (i) Given two vectors $(1,2,1,2)$ and $(0,-1,-1,1)$ in R^4, find all vectors in R^4 that are perpendicular to them. **03**

(ii) Decide which of the following functions on R^2 are inner products, for **04**
$x = (x_1, x_2)$, $y = (y_1, y_2)$

1. $\langle x, y \rangle = 4x_1y_1 + 4x_2y_2 - x_1y_2 - x_2y_1$
2. $\langle x, y \rangle = x_1y_2 - x_2y_1$

(b) Solve the following system of equations using Relaxation method: **07**

$$5x - y - z = 3$$
$$-x + 10y - 2z = 7$$
$$-x - y + 10z = 8$$

OR

Q.5 **(a)** Find the least squares solutions of the system **07**

$$\begin{bmatrix} 1 & 2 \\ 1 & 5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 9 \end{bmatrix}$$

Also determine the orthogonal projection of $[4 \;\; 3 \;\; 9]^T$ in the column space of the given matrix A .

(b) Solve the following system of equations by Crout's method : **07**

$$2x - 6y + 8z = 24$$

$$5x + 4y - 3z = 2$$

$$3x + y + 2z = 16$$

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