Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**ME – SEMESTER-1 (NEW) EXAMINATION – WINTER 2016**

**Subject Code: 2712704** **Date:05/01/2017**

**Subject Name: First Course in Optimization Techniques**

**Time: 2:30 pm to 5:00 pm** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Find the optimum points of the function $f(x) = 4x^3 - 27x$ analytically. Also use Golden Section method to minimize the function $f(x)$ in the interval $(0, 2)$ with $n = 6$ and find true relative error in the optimum point. **07**

**(b)** A furniture company can produce four types of chairs. Each chair is first made in the carpentry shop and then furnished, waxed and polished in the finishing shop. Man-hours required in each are: **07**

| | Chair Type | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Carpentry shop | 4 | 9 | 7 | 10 |
| Finishing shop | 1 | 1 | 3 | 40 |
| Profit per chair (RS.) | 12 | 20 | 18 | 40 |

Total number of man-hours available per month in carpentry and finishing shops are 6000 and 4000, respectively. Assuming abundant supply of raw material and demand for finished products, determine the number of each type of chairs to be produced to maximize profit.

**Q.2** **(a)** Use Powell's method to minimize the function **07**

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

starting from the point $\boldsymbol{X}_1 = [0\ 0]^T$.

**(b)** Maximize $F = 3x_1 + 2x_2 + 5x_3$ **07**

subject to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_2 \leq 420$$
$$x_1 \geq 0, \qquad x_2 \geq 0, \qquad x_3 \geq 0$$

by Revised Simplex Method.

**OR**

**(b)** A company manufactures two kinds of machines, each requiring a different manufacturing technique. The deluxe machine requires 18 hours of labor, 8 hours of testing and yields a profit of Rs.400. The standard machine requires 3 hours of labor, 4 hours of testing and yields a profit of Rs.200. There are 800 hours of labor and 600 hours of testing available each month. A marketing forecast has shown that the monthly demand for the standard machine is to be more than 150. The management wants to know the number of each model to be produced monthly that will maximize total profit. Formulate and solve this linear programming problem graphically. **07**

**Q.3** **(a)** Minimize the function $f(x) = x(x - 1.4)$ in the interval $(0,\ 1)$ to within 10% of the exact value using Interval Halving Method. **07**

**(b)** Consider the following optimization problem: **07**

$$Minimize\ f(X) = (x_1 - 2)^2 + (x_2 - 1)^2$$

subject to

$$2 \geq x_1 + x_2, \qquad x_2 \geq x_1^2.$$

Use Kuhn-Tucker conditions to identify the local minimum from the following vectors, if any:

$$X_1 = [1.5\ \ 0.5]^T, \quad X_2 = [1\ \ 1]^T, \qquad X_3 = [2\ \ 0]^T.$$

**OR**

**Q.3** **(a)** Determine whether the following matrix is positive definite, negative definite or indefinite by finding its eigenvalue: **07**

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}.$$

**(b)** Minimize the function $f(\lambda) = -\lambda\sqrt{4 - \lambda^2}$ using Newton method with $\lambda_1 = 1.0$. Use $\varepsilon = 0.01$ for checking the convergence. **07**

**Q.4** **(a)** ( i ) Find the maxima and minima, if any, of the function **03**

$$f(x) = 4x^3 - 18x^2 + 27x - 7.$$

( ii ) Find the dimensions of an open rectangular box of maximum volume to be manufactured from a given amount of sheet metal of area $S$. **04**

**(b)** Write a short note on classification of optimization problems. **07**

**OR**

**Q.4** **(a)** ( i ) The profit per acre of a farm is given by **03**

$$20x_1 + 26x_2 + 4x_1x_2 - 4x_1^2 - 3x_2^2$$

where $x_1$ and $x_2$ denote, respectively, the labor cost and the fertilizer cost. Find the values of $x_1$ and $x_2$ to maximize the profit.

( ii ) Find the dimensions of a rectangular box with minimum total length of the 12 edges and having volume $V = 1000\ in^3$ using the Lagrange's multiplier method. **04**

**(b)** Describe: Applications of optimization in engineering. **07**

**Q.5** **(a)** ( i ) Write the general form of a constrained optimization problem. **03**

How random search methods are useful for this kind of a problem? **04**

( ii ) Explain Random Jumping Method and Grid Search Method.

**(b)** Write the algorithm of Interior penalty function method. How will you choose the initial value of the penalty parameter? Why the constraints have to be normalized before applying the algorithm? **07**

**OR**

**Q.5** **(a)** ( i ) Transform the following constrained problem into an equivalent unconstrained problem: **03**

$$Minimize\ f(x_1, x_2) = x_1^2 + x_2^2$$

subject to $0 \leq x_1 \leq 1,\ x_2 > 0$.

( ii ) Use Fletcher Reeves' method to minimize the function **04**

$$f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$$

by taking the starting point as $X_1 = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$.

**(b)** Linearize the constraint $4x_1^2 - 3x_1x_2 + 2x_2^2 - 1 \leq 0$ at the point $X_1 = [-1\ \ 1]^T$. Also write the Sequential Linear Programming algorithm for constrained optimization. **07**

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