Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER II– EXAMINATION – WINTER - 2016

Subject Code: 2720501 **Date: 17/11/ 2016**

Subject Name: Statistical Signal analysis

Time: 2:30 pm to 5:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** With example state the uses of the Probability Density Function (PDF). Explain how it can be derived. Show that the CDF is monotonically non-decreasing function **07**

(b) A communication channel accept an arbitrary voltage input v and outputs a voltage Y= v+N, where N is a Gaussian random variable with mean 0 and variance of 1. Suppose that the channel is used to transmit binary information as follows: **07**

To transmit 0 $\rightarrow$ input -1

To transmit 1 $\rightarrow$ input 1

The receiver decides a 0 was send if voltage is negative and a 1 otherwise find probability of the receiver making an error if a 0 was sent, is a 1 was sent

Q.2 **(a)** Explain the central limit theorem with proper example and give proof the theorem **07**

(b) Let X be a continuous random variable with PDF **07**

$$f_x(x) = \begin{cases} kx & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

(a) Determine the value of k and sketch fx(x).

(b) Find and sketch corresponding CDF Fx(x).

(c) Find P(¼ < X ≤ 2)

OR

(b) The joint PDF of bivariate random variable (x,y) is given by, **07**

$$f_{XY}(xy) = \begin{cases} kxy & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$$

Where k is a constant. Find

(a) Find k.

(b) Are X and Y are statistically independent random variable.

(c) Find P(X+Y<1)

Q.3 **(a)** Let Φ be a uniformly distributed random variable in the interval - π/2 ≤ Φ ≤ π/2. Suppose random variable X is defined as, **07**

X=a tan[Φ]

What will be the distribution of X?

(b) For conditional expectation, Find E[E[Y|X]]. Where E[.] is expected operator **07**

OR

Q.3 **(a)** In mobile radio communication when any obstruction comes in to the path of signal then the region behind obstruction is called shadow region, if transmitted **07**

signal random variable is X and in shadow region received variable is Y, then X and Y is related as $Y=e^X$

If transmitted signal has Gaussian distribution with mean mx and variance σ^2 then find distribution of received signal in shadow region.

(b) Define second order initial and central Moment. What is the significance of Second order central moment in Statistical domain. **07**

Find first order central moment of random variable X. comment on the result.

Q.4 **(a)** Two Company's share price fluctuation can be modelled as Random Process with following parameters **07**

Company -A Share price (in Rs.)Random Signal has

DC value of 20 . (2) RMS value is 40 . (3) Xr1(t) and Xr1(t+ τ) are independent for $|\tau| \leq 100$ days. (4) R(τ) decreases linearly with $|\tau|$ for $0 \leq \tau \leq 100$ days.

Company -B Share price (in Rs.) Random Signal has

DC value of 15 . (2) RMS value is 100 . (3) Xr1(t) and Xr1(t+ τ) are independent for $|\tau| \leq 60$ days. (4) R(τ) decreases linearly with $|\tau|$ for $0 \leq \tau \leq 60$ days.

Plot its autocorrelation function of both the random signals and as share investor advisor, advice your client about investment in company A and company B.

(Note: Exact plot is not required but draw plot based on given parameters of random signal)

(b) Sketch the ensemble of the random process **07**

$$x(t) = a\cos(wt + \theta)$$

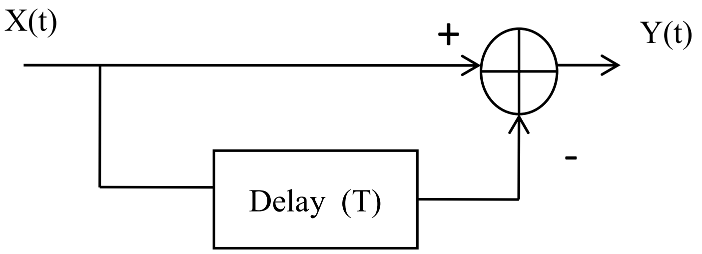
where w and θ are constants and a is an RV uniformly distributed in the range (-A,A).

(a) Just by observing the ensemble, determine whether this is a stationary or a non stationary process.

(b) Determine $\overline{x(t)}$ and Rx(t1,t2) for this random process and determine whether this is a wide-sense stationary process.

OR

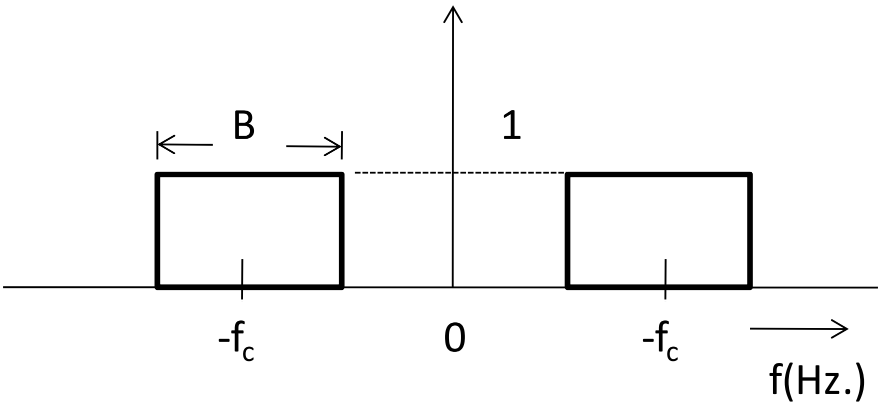
Q.4 **(a)** Suppose that a Wide sence stationary process X(t) with power spectrum Sxx(W) is the input to the filter shown in figure below. Find power spectrum of the output process Y(t) **07**



(b) Consider a random process X(t)=U cos t + V sin t, where U and V are independent random variable each of which assumes the value -2 and 1 with the probability 1/3 and 2/3 respectively. Show that X(t) is Wide Sense Stationary but not Strict Sense Stationary **07**

Q.5 **(a)** The autocorrelation function of a stochastic process X(t) is $\phi_{xx}(\tau) = 0.5N_o\ \delta(\tau)$. Such process is called white process. **07**

If X(t) is the input to an idle band pass filter having the frequency response as shown in figure below. Determine the total noise power at the output of filter.



(b) With proper example explain the wide sense stationary process and Ergodic random process **07**

OR

Q.5 **(a)** Are the following covariance functions are valid covariance functions of a real stationary process? Given answer with proper reasons. **07**

(a) $R(\tau) = \cos(t)$ (b) $R(\tau) = \begin{cases} \tau & |\tau| < 1 \\ 0 & |\tau| > 1 \end{cases}$

(b) Define **07**

1. Almost sure convergence
2. Mean square sense convergence.

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