Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER-1 (OLD) EXAMINATION – WINTER 2016

Subject Code: 710401N **Date:17/11/2016**

Subject Name: Statistical Signal Analysis

Time:10:30 Am to 1:00 Pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	Describe the statistical approach and axiomatic approach to a theory of probability. Explain the concept of random variable and random process.	**07**
	(b)	State and prove central limit theorem.	**07**
Q.2	**(a)**	Define Cumulative Distribution Function. State and prove all the properties of CDF.	**07**
	(b)	Explain the conditional probability with suitable example. A lot of 100 semiconductor chips contain 20 that are defective. Two chips are selected at random, without relacement, from the lot. (a) What is the probability that the first one selected is defective? (b) What is the probability that the second one selected is defective given that the first one was defective? (c) What is the probability that both are defective?	**07**
		OR	
	(b)	Two events A and B are mutually exclusive. Can they be independent? Let two numbers x and y are selected at random between zero and one. Let the events A,B and C be defined as A={x>0.5}, B={y>0.5} and C={x>y}. Are the events A and B independent? Are A and C independent?	**07**
Q.3	**(a)**	State and prove Markov and Chebyshev inequalities.	**07**
	(b)	Define characteristic function. Find the characteristic function of the uniform random variable in the interval [a,b]. Find the mean and variance of uniform random variable by applying the moment theorem.	**07**
		OR	
Q.3	**(a)**	What is conditional expectation? Determine E[E[Y|X]].	**07**
	(b)	Let X be a continuous random variable with PDF $$f_X(x) = \begin{cases} kx & 0 < x < 1 \\ 0 & otherwise \end{cases}$$ (a) Determine the value of k and sketch $f_X(x)$ (b) Find and sketch corresponding CDF $F_X(x)$ (c) Find $P(1/4<X<2)$.	**07**
Q.4	**(a)**	Derive the equation of marginal PDF. Find the normalization constant c and the marginal PDF's for the following joint PDF: $$f_{X,Y}(x,y) = \begin{cases} ce^{-x}e^{-y} & 0 \le y \le x < \infty \\ 0 & elsewhere \end{cases}$$	**07**
	(b)	Define correlation and covariance of two random varibles. Given X = cosθ and Y = sinθ, where θ is an R.V. uniformly distributed in the range (0,2π), show that x and y are uncorrelated but are not independent.	**07**
		OR	
Q.4	**(a)**	What is the law of large numbers? Describe different laws of large number.	**07**

(b) Let ζ be selected of random from the interval S = [0,1], and let the probability that ζ is in a subinterval of S be given by the length of the subinterval. Define the following sequences of random variables for $n \geq 1$; **07**

$$X_n(\zeta) = \zeta^n, \qquad Y_n(\zeta) = \cos^2 2\pi\zeta, \qquad Z_n(\zeta) = \cos^n 2\pi\zeta$$

Do the sequences convergence, and if so, in what sense and to what limiting random variable?

Q.5 **(a)** Classify the random processes and explain each in detail. **07**

(b) Sketch the ensemble of the random process $x(t) = a \cos(\omega t + \theta)$, where ω and θ are constants and a is an R.V. uniformly distributed in the range (-A,A). Just by observing the ensemble, determine if this is a stationary or nonstationary process. Also determine whether this is a wide-sense stationary process. **07**

OR

Q.5 **(a)** Explain the concept of mean square derivatives of a random process with necessary equations. **07**

(b) Determine the PSD and the mean square value of a random process $x(t) = A \cos(\omega t + \theta)$, where A and ω are constants and θ is an R.V. uniformly distributed in the interval $(0,2\pi)$. **07**

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