Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER-1(OLD COURSE) EXAMINATION – WINTER 2016

Subject Code: 710703N **Date: 19/11/2016**

Subject Name: MODERN CONTROL SYSTEMS

Time:10:30 Am to 1:00 Pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define and discuss the concept of state, state variables, state vector, state space and state equations. **07**

(b) Write and prove the properties of the State Transition Matrix (STM). **07**

Q.2 **(a)** Obtain the transfer function of the system defined by **07**

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(b) Consider the following transfer function system: **07**

$$\frac{Y(s)}{U(s)} = \frac{s+6}{s^2+5s+6}$$

Obtain the state space representation of this system in (a) Controllable canonical form and (b) Observable canonical form

OR

(b) Compute e^{At} of the following matrix. **07**

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Q.3 **(a)** Show that the following system is not completely observable: **07**

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Where,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix}$$

(b) Derive the condition for checking the Controllability for a given system. **07**

OR

Q.3 **(a)** Prove that the Eigen values are invariant under a linear transformation. **07**

(b) Is the following system completely state controllable? **07**

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 20 & 9 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q.4 **(a)** Explain the variable gradient method for the determination of liapunov function. **07**

(b) Obtain the time response of the following system : **07**

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Where u(t) is the unit step function occuring at t=0 or, $u(t) = 1(t)$

OR

Q.4 **(a)** Define the Sylvester's criterion for checking the definiteness and also check the definiteness of the following functions **07**

$V(x) = x_1^2 + 2x_2^2$

$V(x) = 10x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_1x_3$

(b) Explain the principle of duality that clarify apparent analogy between controllability and observability. **07**

Q.5 **(a)** Design a state feedback controller gain using pole placement technique for the system given by **07**

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

The desired pole locations of the closed loop system are *s* = -3 and *s* = -5

(b) Express matrix A as a sum of symmetric and a skew symmetric matrix where, $A = \begin{bmatrix} 2 & 3 & -3 \\ 5 & 6 & -6 \\ -7 & 0 & -9 \end{bmatrix}$ **07**

OR

Q.5 **(a)** Explain design of control system with observers. Briefly discuss both Configurations i.e. observer controller in feed forward path and in feedback path. **07**

(b) Prepare a technical note on "Application of Kalman's test" **07**

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