Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME – SEMESTER III (OLD) – EXAMINATION – WINTER-2016

Subject Code: 730403 **Date:25/10/2016**

Subject Name: Optimization Techniques

Time:02:30 pm to 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Write the classification of optimization problems along with the methods to solve each type of problem. **07**

(b) (i) Define the terms: convex function, unimodal function, gradient of a function, behavioral constraint **04**

(ii) Write the necessary and sufficient condition for a function of one variable to have an optimum at a point. **03**

Q.2 **(a)** Use Cauchy's steepest descent method to minimize the function **07**

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

starting from the point $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Perform two iterations.

(b) Write the algorithm of Random Walk Method. **07**

OR

(b) Maximize $F = x_1 + 2x_2 + x_3$ subject to **07**

$$2x_1 + x_2 - x_3 \leq 2,$$
$$-2x_1 + x_2 - 5x_3 \geq -6,$$
$$4x_1 + x_2 + x_3 \leq 6,$$
$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

using Revised Simplex Method.

Q.3 **(a)** A manufacturing firm produces two machine parts using lathes, milling machines and grinding machines. The different machining times required for each part, the machining times available on different machines, and the profit on each machine part are as follows: **07**

Type of machine	Machining time required (min)		Max. time available/ week (min)
	Machine Part I	Machine Part II	
Lathes	10	5	2500
Milling machines	4	10	2000
Grinding machines	1	1.5	450
Profit per unit	50	100	

Use Simplex method to determine the number of parts I and II to be manufactured per week to maximize the profit.

(b) Use Secant Method to minimize the function **07**

$$f(\lambda) = 0.65 - \frac{0.75}{1+\lambda^2} - 0.65\,\lambda\, tan^{-1}\frac{1}{\lambda}$$

with an initial step size of $t_0 = 0.1, \lambda_1 = 0.0$, and $\varepsilon = 0.01$.

OR

Q.3 **(a)** Use Fibonacci Method with $n = 6$ to maximize the function **07**

$$f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda$$

in the interval $[0, 3]$.

(b) Apply Interval Halving Method to find the minimum of **07**

$$f(x) = x(x - 1.5)$$

in the interval $(0.00, 1.00)$ to within 10% of the exact value.

Q.4 **(a)** (i) Find the minimum of the function $f(x) = (x-2)^3$, if exists. **03**

(ii) The profit per acre of a farm is given by **04**

$$20x_1 + 26x_2 + 4x_1x_2 - 4x_1^2 - 3x_2^2$$

where x_1 and x_2 denote, respectively, the labor cost and fertilizer cost. Find the values of x_1 and x_2 to maximize the profit.

(b) Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. **07**

OR

Q.4 **(a)** (i) Determine whether the following matrix is positive definite, negative definite or indefinite by evaluating the signs of its submatrices. **03**

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}.$$

(ii) Consider the following optimization problem: **04**

Maximize $f = -x_1 - x_2$

subject to

$$x_1^2 + x_2 \geq 2, \quad 4 \leq x_1 + 3x_2, \quad x_1 + x_2^4 \leq 30.$$

Find whether the design vector $\boldsymbol{X} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ satisfies the Kuhn-Tucker conditions for a constrained optimum. What are the values of the Lagrange's multipliers at the given design vector?

(b) Find the maximum of the function $f(\boldsymbol{X}) = 2x_1 + x_2 + 10$ subject to $g(\boldsymbol{X}) = x_1 + 2x_2^2 = 3$ using the Lagrange multiplier method. Also find the effect of changing the right-hand side of the constraint on the optimum value of f. **07**

Q.5 **(a)** Apply Powell's method to minimize **07**

$$f(x_1, x_2) = 6x_1^2 - 6x_1x_2 + 2x_2^2 - x_1 - 2x_2$$

from the starting point $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Take $\varepsilon = 0.01$.

(b) Explain sequential linear programming method with a simple example. **07**

OR

Q.5 **(a)** Perform two iterations of Univariate Method to minimize **07**

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

from the starting point $\boldsymbol{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$. Take $\varepsilon = 0.01$.

(b) How Penalty function Method is useful in optimization? What is the difference between Interior Penalty Function Method and Exterior Penalty Function Method? What are the familiar penalty functions? **07**

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