Seat No.: ________ Enrolment No.____________

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: 710301N **Date: 08-01-2013**
Subject Name: Control Engineering
Time: 02.30 pm – 05.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q-1 **(a)** Explain Full order Observer in detail. **08**

(b) Derive the equation of fundamental necessary condition of variational calculus for the given trajectory, x = x^*(t), such that the integral , J(x) = $\int_{t_0}^{t_1} g(x,\dot{x},t)$dt ,along the trajectory has a relative extremum, considering fixed end points condition. **06**

Q.2 **(a)** Prove that the system with state model, $\dot{x}(t) = Ax(t) + Bu(t)$ and y(t)=Cx(t) +Du(t) is asymptotically stable if and only if all the eigenvalues of matrix A have negative real parts. **08**

(b) Find the curve with minimum arc length between the point x(0) = 1 and the line t_1 = 4. **06**

OR

(b) Prove that the nonlinear system $\dot{x} = f(x)$; f(0) = 0 , is asymptotically stable at the origin if there exist a constant, positive definite, symmetric matrix P such that the matrix F(x) = J^T(x)P +PJ(x) is negative definite for all x. Here V(x) = f^TPf . **06**

Q.3 **(a)** Consider the linear autonomous system **09**

X(k+1) = $\begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix}$ x(k)

Using direct method of Lyapunov, determine stability of the equilibrium state.

(b) What is Dynamic programming? State and explain principle of causality and principle of invariant imbedding for optimal control system. **05**

OR

Q.3 **(a)** The system is represented by $\dot{x} = Ax + Bu$, where, **09**

A = $\begin{bmatrix} 0 & 0 & -3 \\ 2 & 0 & -7 \\ 0 & -1 & 0 \end{bmatrix}$; B = $\begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix}$

Obtain equivalent system in controllable companion form.

(b) Prove that the linear system, $\dot{x} = Ax$ is globally asymptotically stable at the origin if and only if for any given symmetric positive definite matrix Q, there exist a symmetric positive definite matrix P that satisfies the matrix equation, A^T P + PA = -Q **05**

Q.4 **(a)** Draw the flow chart of dynamic programming algorithm for optimal control system. **04**

	(b)	Derive the equation of state feedback control law for the discrete time linear state regulator system.	**10**
		OR	
Q.4	**(a)**	Explain the Negative Exponential method for finding out the numerical solution of Riccati equation.	**04**
	(b)	Find the extremal of the functional $J(x) = \int_0^{\pi/4} (x_1^2 + \dot{x}_2^2 + \dot{x}_1 \dot{x}_2)\, dt$ The boundary conditions are $x_1(0) = 0$, $x_1(\pi/4) = 1$, $x_2(0) = 0$, $x_2(\pi/4) = -1$	**10**
Q.5	**(a)**	Explain the response of linear continuous time system to white noise with necessary derivations.	**11**
	(b)	Define: Random vector process Wide-sense stationary process Discrete white noise	**03**
		OR	
Q.5	**(a)**	Prove that if the system, $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + Du(t)$ is controllable and b_i ($\neq 0$) is the ith column of B, then there exist a feedback matrix K_i such that the single-input system $\dot{x} = (A+BK_i)x + b_i r_i$ is controllable.	**07**
	(b)	Explain stochastic optimal linear regulator system in detail.	**07**

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