Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: 710901N **Date: 08-01-2013**

Subject Name: Theory of Elasticity

Time: 02.30 pm – 05.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Show that according to linear strain theory, the volumetric strain is equal to the sum of three linear strains. **07**

(b) For the plane state of stress conditions determine the values of two principal stresses and maximum shear stress. **07**

Q.2 **(a)** For a circular rod subjected to a torque, the displacement components at any point (x,y,z) are obtained as **07**

$Ux = -\tau yz + ay + bz + c$
$Uy = \tau xz - ax + ez + f$
$Uz = -bx - ey + k$

Where a, b, c, e, f and k are constants and τ is the shear stress.
Select the constants a, b, c, e, f and k such that at the end section z=0 is fixed in the following manner

(i) Point *o* has no displacement.
(ii) The element Δz of the axis rotates neither in the plane xoz nor in plane yoz
(iii) The element Δy of the axis does not rotate in the plane xoy.

Also determine the strain components.

(b) Evaluate the following statements:

(i) The six strain components for a given state of strain at a point must obey six compatability conditions. **03**

(ii) It is observed that for the bulk modulus to be positive, the value of Posson's ratio cannot exceed the value of 0.5. **04**

OR

(b) Justify the following statements:

(i) A small sphere when immersed in fluid experiences an isotropic stress. **03**

(ii) Principal planes corresponding to a given state of stress at a point are mutually orthogonal. **04**

Q.3 **(a)** Compute the Lame's constant λ and μ for **07**

(i) Steel having $E = 207 \times 10^6$ kPa and $\nu = 0.3$
(ii) Concrete having $E = 28 \times 10^6$ kPa and $\nu = 0.2$

Compare the values of Lame's constant λ and μ for steel and concrete and comment on the same.

(b) Define bulk modulus K for an isotropic material and show that it is related to Lame's constant λ and μ by the following relationship $K = (3\lambda + 2\mu)/3$. **07**

OR

Q.3 **(a)** State and prove the Castlingo's second theorem. **07**

(b) Prove that the partial differential coefficient of the strain energy function with reference to force F_r gives the displacement corresponding to it. **07**

Q.4 **(a)** Consider a thick walled cylinder subjected to internal pressure and derive an expression for radial and circumferential stress for a plane stress case. **07**

(b) Derive an expression for radial and circumferential stress for a thin and solid rotating circular disc. **07**

OR

Q.4 **(a)** Consider a thick walled cylinder subjected to external pressure and derive an expression for radial and circumferential stress for a plane stress case. **07**

(b) Derive an expression for radial and circumferential stress for a thin rotating circular disc having a hole of radius 'a'. **07**

Q.5 **(a)** Consider uniformly rotating long solid circular shaft and derive an expression for radial, axial and circumferential stress for a plane strain case. **07**

(b) A solid steel propeller shaft, 60 cm in diameter is rotating at the speed of 300 rpm. If the shaft is constrained at its ends so that it cannot expand or contract longitudinally, calculate the total longitudinal thrust over a cross section due to rotational stresses. Consider Poisson's ratio as 0.3 and the weight of the steel as 0.07938 N/cm^3. **07**

OR

Q.5 **(a)** Consider uniformly rotating hollow circular shaft and derive an expression for radial, axial and circumferential stress for a plane strain case. **07**

(b) A flat steel disk of 75 cm outside diameter with a 15 cm diameter hole is shrunk around a steel shaft. Consider a shrink fit allowance of 0.0075 cm in radius and E as 214×10^6 kPa. Determine the stresses due to shrink fit. **07**

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