Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: 712901N **Date: 08/01/2013**

Subject Name: Mathematics for Researcher

Time: 02.30 pm – 05.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Obtain Fourier series for the function $f(x) = x + x^2$ in the interval $[-\pi, \pi]$. Hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ... = \frac{\pi^2}{6}$. **07**

(b) Define rounding off and explain rules to round off a number correct to n significant figures. **07**

Q.2 **(a)** Evaluate following integral by using Simpson's $1/3$ rule $\int_4^{5.2} \log_e x \, dx$. **07**

(Take h=0.1)

OR

(b) Detemine largest eigen value and corresponding eigen vectors of the following matrix correct to three decimal places by using power method. $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ **07**

OR

(b) Round-off the number 865250.5 up to four significant figures and compute absolute, relative and percentage errors. **07**

Q.3 **(a)** A curve is drawn to pass through the points given by the following table. **07**

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Find the area bounded by the curve, x-axis and the lines $x = 1$ and $x = 4$.

(b) Find $y'(0)$ and $y''(0)$ from the following table. **07**

x	0	1	2	3	4	5
y	4	8	15	7	6	2

OR

Q.3 **(a)** Express the function $f(x) = x + |x|$ as a Fourier series in the interval $[-\pi, \pi]$ **07**

(b) Find the Fourier transform of the function $f(x) = \begin{cases} xe^{-x}; & x > 0 \\ 0 \quad ; & x < 0 \end{cases}$ **07**

Q.4 **(a)** Solve the following system of equations by using Gauss elimination method. **07**

$$5x + y + z + w = 4$$
$$x + 7y + z + w = 12$$
$$x + y + 6z + w = -5$$
$$x + y + z + 4w = -6$$

(b) State Cayley-Hamilton theorem and using it find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ **07**

OR

Q.4 **(a)** By using Gauss-Jordan method find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ **07**

(b) Explain Escalator (Partition) method. **07**

Q.5 **(a)** Using Taylor's series method compute y(0.1) correct to five decimal places, given that $\frac{dy}{dx} = x^2 y - 1$, y(0)=1. **07**

(b) Apply Runge-Kutta method of order four to find an approximate value of y when x=0.1, given that $\frac{dy}{dx} = x + y^2$, y(0)=1 **07**

OR

Q.5 **(a)** Find a real root of the equation $x^4 - x - 10 = 0$ correct to three decimal places by using Newton-Raphson method. **07**

(b) Find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places by using bisection method. **07**

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