Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: 714704N **Date: 12-01-2013**

Subject Name: Optimization Theory and Practice

Time: 02.30 pm – 05.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q:1 Explain the interior penalty and exterior penalty optimization techniques for single variable function $f(X) = \alpha x_1$, subjected to $g_1(X) = \beta - x_1$. **14**

Q:2 **(a)** Explain the various steps of Random Walk Method used to optimize non linear unconstrained problem. **07**

(b) Draw the flow chart for Simulated annealing algorithm. **07**

OR

(b) What are the basic operations used in Genetic algorithm? What is the fitness function in Genetic algorithm? **07**

Q:3 **(a)** Discuss the detailed classification of optimization problems with suitable examples. **07**

(b) A beam of uniform rectangular cross section is to be cut from a log having a circular cross section of diameter 50 cm. The beam has to be used as a cantilever beam (length is fixed) to carry a concentrated load at free end. Find the dimension of the beam that corresponds to the maximum bending stress capacity. Use Lagrange Multipliers technique for optimization. **07**

OR

Q:3 **(a)** The potential energy (U) for spring mass system is defined by the following equation: **07**

$$U = \left[\frac{1}{2}k_2x_1^2 + \frac{1}{2}k_3\left(x_2 - x_1\right)^2 + \frac{1}{2}k_1x_2^2\right] - Px_2$$

where, k_i = spring stiffness (i =1,2,3), and P is applied external load. Find the displacement x_1 and x_2 of masses m_1 and m_2, respectively. Use the principle of minimum potential energy.

(b) Explain the significance of saddle points. Find the saddle points for the following function: **07**

$$f\left(x_1, x_2\right) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$$

Q:4 **(a)** Find whether the design vector $X = \{1, 1\}^T$ satisfies the Kuhn–Tucker conditions for a constrained optimum for the following function. **07**

Maximize $f = -x_1 - x_2$

subject to

$$x_1^2 + x_2 \geq 2$$

$$4 \leq x_1 + 3x_2$$

$$x_1 + x_2^4 \leq 30$$

(b) Explain the following terms related to Linear Programming Problem:

1. Unboundness of solution **02**
2. Degeneracy of solution **02**
3. Dual form of LPP **03**

OR

Q:4 **(a)** A retired person wants to invest upto amount of Rs 30,000 in fixed income securities. His broker recommends investing in two bonds: Bond A yielding 7% and Bond B yielding 10%. After some consideration, he decides to invest at most Rs 12,000 in Bond b and atleast Rs 6000 in Bond A. He also wants the amount invested in Bond A to be atleast equal to the amount invested in Bond B. What should the broker recommend if the investor wants to maximise his return on investment? Solve graphically. **07**

(b) Find the minimum of the function **07**

$$f(x) = 0.65 - \frac{0.75}{1+x^2} - 0.65x\tan^{-1}\frac{1}{x}$$

Using interval halving method in the interval [0,1] using five steps.

Q:5 **(a)** Find the maximum of the following function in the interval [0.75, 0.8] using Golden section method in four steps. **07**

$$f(x) = \frac{0.5}{\sqrt{1+x^2}} - \sqrt{1+x^2}\left(1 - \frac{0.5}{1+x^2}\right) + x$$

(b) Find the minimum of function f(x) = x / log x using Newton's method for accuracy of 0.001 and initial value $x_1 = 2$. **07**

OR

Q:5 Perform three iterations of steepest descent method and univariate method to minimize the following function from starting point $x_1 = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$. **14**

$$f = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1 + 10$$

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