Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – III • EXAMINATION – WINTER 2012

Subject code: 730403 **Date: 26/12/2012**

Subject Name: Optimization Techniques

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Determine the maximum and minimum values of the function $f\left(x\right)=12x^5-45x^4+40x^3+5.$ **03**

(ii) Show that the right circular cylinder of given surface (including the ends) and maximum volume is such that its height is equal to the diameter of the base. **04**

(b) (i) consider the following problem: **03**

Minimize $f\left(X\right)=x_1^2+x_2^2+x_3^2$ subject to

$x_1+x_2+x_3\geq 5, \qquad 2-x_2x_3\leq 0,$

$x_1\geq 0,\ x_2\geq 0,\ x_3\geq 2.$

Determine whether the Kuhn-Tucker conditions are satisfied at $X=\left[2,1,2\right]^T$.

(ii) Find the solution of the following problem graphically: **04**

Maximize $f=15x+10y$ subject to

$4x+6y\leq 360,\ \ 3x\leq 180,$

$5y\leq 200,$

$x\geq 0,\ \ y\geq 0.$

Q.2 **(a)** Solve the following LP problem by Simplex Method: **07**

Maximize $f=4x+3y$ subject to

$2x+y\leq 1000, \qquad x+y\leq 800,$

$x\leq 400, \qquad y\leq 700, \qquad x,y\geq 0.$

(b) A company has three production facilities S_1,S_2 and S_3 with production capacity of 7, 9 and 18 units per week of a product, respectively. These units are to be shipped to four warehouses D_1,D_2,D_3 and D_4 with requirement of 5, 8, 7 and 14 units per week, respectively. The transportation costs per unit between factories to warehouses are given in the table below: **07**

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	50	45	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Minimize the total transportation cost.

OR

	(b)	Use the revised simplex method to solve the following LP problem: Maximize $f = 2x_1 + x_2$ subject to the constraints $3x_1 + 4x_2 \le 6,$ $6x_1 + x_2 \le 3,$ $x_1, x_2 \ge 0.$	**07**
Q.3	**(a)**	Find the minimum of $f = x(x-1.5)$ in the interval $(0,1)$ to within 10% of exact value using Interval Halving method.	**07**
	(b)	Minimize $f = \lambda^5 - 5\lambda^3 - 20\lambda + 5$ in the interval $[0,5]$ by the Fibonacci method using $n = 6$.	**07**
		OR	
Q.3	**(a)**	Minimize $f = \lambda^5 - 5\lambda^3 - 20\lambda + 5$ in the interval $[0,5]$ by the Golden Section method using $n = 6$.	**07**
	(b)	Find the minimum of the function $f(\lambda) = 0.65 - \frac{0.75}{1+\lambda^2} - 0.65\lambda \tan^{-1}\left(\frac{1}{\lambda}\right)$ using the secant method with an initial step size of $t_0 = 0.1, \lambda_1 = 0.0,$ and $\varepsilon = 0.01.$	**07**
Q.4	**(a)**	What are pattern directions? Describe the algorithm to minimize a function with n variables having no constraints by using Powell's method.	**07**
Q.4	**(b)**	Minimize $f(x_1, x_2) = 2x_1^2 + x_2^2$ from the starting point $X_1 = [1,2]^T$ using Steepest Descent method. Perform only two iterations.	**07**
		OR	
Q.4	**(a)**	Describe the general procedure to minimize a nonlinear function with n variables having no constraints by Hooke and Jeeves' method.	**07**
	(b)	Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_2$ from the starting point $X_1 = [0,0]^T$ using Fletcher-Reeves' method. Iterate until $\lvert\nabla f\rvert < 0.001.$	**07**
Q.5	**(a)**	Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ from the starting point $X_1 = [0,0]^T$ using Newton's method.	**07**
	(b)	Explain the basic approach of the penalty function method.	**07**
		OR	
Q.5	**(a)**	Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ from the starting point $X_1 = [0,0]^T$ using Univariate method. Perform only two iterations.	**07**
	(b)	Convert the following constrained optimization problem into unconstrained one by making change of variables and then solve it: Find the dimensions of a rectangular prism type box that has the largest volume when the sum of its length, width and height is limited to a maximum value of 60 in. and its length is restricted to a maximum value of 36 in.	**07**

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