

GUJARAT TECHNOLOGICAL UNIVERSITY
M. E. - SEMESTER – I • EXAMINATION – WINTER • 2013

Subject code: 710107N**Date: 03-01-2014****Subject Name: Quantum Theory & Algorithm Design****Time: 10.30 am – 01.00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Answer the following. **14**

- (i) Define: Qubit, Vector Space, Outer Product.
- (ii) Compare Recursion with Iteration with example.
- (iii) Explain why analysis of algorithms is important? Explain: Worst Case, Best Case & Average Case Complexity.
- (iv) When a binary relation is anti-symmetric? Give example of a relation that is reflexive, transitive but anti-symmetric.

Q.2 (a) (i) Find eigen values of an operator with matrix representation **07**

$$A = \begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix}$$

- (ii) For the following qubits, if a measurement is made, what is the probability that we find the qubit in state $|0\rangle$ and state $|1\rangle$?

$$|x\rangle = \frac{(1+i)}{\sqrt{3}} |0\rangle - \frac{i}{\sqrt{3}} |1\rangle$$

(b) Explain Krushkal's Algorithm to find Minimum Spanning Tree with illustration. **07****OR**(b) Explain Prim's Algorithm to find Minimum Spanning Tree with illustration. **07****Q.3** Answer the following.(Any TWO) **14**

(a) Suppose

$$|\psi\rangle = 3|0\rangle - 2i|1\rangle, |\phi\rangle = |0\rangle + 5|1\rangle$$

- (i) Show that these states obey the Cauchy-Schwarz and triangle inequalities.

- (ii) Normalize the states.

(b) Suppose that a system is in the state

$$|\psi\rangle = \frac{\sqrt{2}}{\sqrt{3}} |0\rangle - \frac{1}{\sqrt{3}} |1\rangle$$

- (i) Compute $\text{Tr}(\rho)$ and $\text{Tr}(\rho^2)$. Is this a mixed state ?
- (ii) Find $\langle X \rangle$ for this state.

(c) Explain the characteristics of Greedy Algorithms. Compare Dynamic Programming Method with Greedy Algorithm with illustration.

Q.4 Answer the following.(Any TWO) **14**

- (a) Solve the following 0/1 Knapsack Problem using Dynamic Programming Method. Write the equation for solving the problem.

		n = 5, W = 11				
Object	→	1	2	3	4	5
Weight (w)	→	1	2	5	6	7
Value (v)	→	1	6	18	22	28

- (b) Use Gram-Schmidt process to construct an ortho-normal basis set from following Vectors.

$ v_1\rangle = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$	$ v_2\rangle = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$	$ v_3\rangle = \begin{pmatrix} 3 \\ -7 \\ 1 \end{pmatrix}$
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- (c) A three state system is in the state

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

Write down necessary projection operators and calculate the probabilities $\text{Pr}(0)$, $\text{Pr}(1)$ and $\text{Pr}(2)$

Q.5 Answer the following.(Any TWO)

14

- (a) (i) What do you mean by Trace? Give important properties of Trace. How do you find trace of an operator?
(ii) Explain all Pauli Operators and significance of them.
- (b) Explain the use of Divide & Conquer technique for Heap Sort Method with illustration.
- (c) Give and Explain the Breadth First Search and Depth First Search Algorithm for graph searching with illustration.
