Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2013

Subject code: 710703N **Date: 03-01-2014**

Subject Name: Modern Control System

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

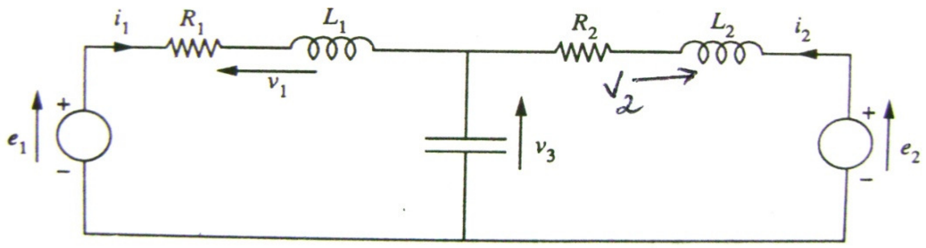
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Answer the following: **07**

(1) Explain the properties of State Transition Matrix.

(2) Compare state space technique and transfer function technique.

(b) Obtain the state space model for the circuit shown below. Assume e_1, e_2 are inputs and v_1, v_2 *and* v_3 are outputs. **07**



Q.2 **(a)** Determine eigenvalues of matrix A= $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$. **07**

Prove that eigenvalues are invariant under a linear transformation.

(b) Which are the various canonical forms in which state representation can be made? Obtain state representation in all possible canonical forms for a system given by $\frac{Y(s)}{U(s)} = \frac{s+3}{s^2+3s+2}$. **07**

OR

(b) Determine the time response of a system represented by , **07**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u .$$

Where $u(t)$ is a unit step function occurring at $t = 0$ *and*

$$x^T(0) = [1 \quad 0].$$

Q.3 **(a)** The state space model of a system is given below. **09**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -160 & -56 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -14 \end{bmatrix} u .$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Derive a suitable transformation by which another state space model of the same system can be written as :

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -14 & -56 & -160 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u .$$

(b) Determine using Cayley-Hamilton theorem: **05**

$$f(A) = A^9 \ for\ A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}.$$

OR

Q.3 **(a)** Derive state representation in Jordan's canonical form from a generalized transfer function in which the denominator polynomial involves multiple roots. **07**

(b) Compute e^{At} using any two different analytical methods where **07**

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}.$$

Q.4 **(a)** Develop Principle of Duality between controllability and observability. **07**

(b) A single input single output system is given as **07**

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t)\ , \qquad and$$

$$y(t) = \begin{bmatrix} 2 & 0 & 5 \end{bmatrix} x(t).$$

Check whether it is controllable or not.

OR

Q.4 **(a)** Explain the concept of controllability. Discuss Gilbert's method to test the controllability. Also discuss its limitations. **07**

Q.4 **(b)** A single input single output system is given as **07**

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} u(t)\ , \qquad and$$

$$y(t) = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} x(t).$$

Check whether it is observable or not.

Q.5 **(a)** Explain Liapunov's stability criterion for the linear time invariant system. **07**

(b) A regulator system is given by – **07**

$$\dot{x} = Ax + Bu\ ,$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The system uses the state feedback control $u = -kx$.
The desired pole locations of close loop system are
$s = -2 \pm j4\ and\ s = -10.$ Using direct substitution method, design a state feedback controller gain.

OR

Q.5 **(a)** Discuss necessary and sufficient condition for state observation and hence explain the design procedure of a full state observer. **07**

(b) Using Liapunov's direct method determine the stability of the system **07**

$\dot{x} = Ax$, where $A = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix}.$

****