Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2013

Subject code: 710901N Date: 23-12-2013

Subject Name: Theory of Elasticity

Time: 10.30 am – 01.00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** "The three values of principal stresses are always real and not imaginary". Comment on the degree of the validity of a given statement with a mathematical proof. **07**

(b) Derive the Lame's displacement equations of equilibrium. **07**

Q.2 **(a)** Write a Hooke's law and extend this concept to derive generalized Hooke's law for relating six strain components with stress components assuming linear variation of stress with strain for homogeneous material. Using this law derive a stress strain relationship for linear, elastic, homogeneous and isotropic material. **07**

(b) The state of stress characterized by τ_{ij} is given below. Resolve the given state into a hydrostatic state and a pure shear state. Determine the normal and shearing stresses on an octahedral plane. Compare these with the σ_{oct} and τ_{oct} calculated for the hydrostatic and the pure shear states. Are the octahedral planes for the given state, the hydrostatic state and the pure shear state the same or are they different? Explain why? **07**

$$\tau_{ij} = \begin{pmatrix} 10 & 4 & 6 \\ 4 & 2 & 8 \\ 6 & 8 & 6 \end{pmatrix}$$

OR

(b) The displacement field in micro units for a body is given by **07**

$$u = \left(x^2 + y\right)i + \left(3 + z\right)j + \left(x^2 + 2y\right)k$$

Determine the principal strains at (3,1,-2) and the direction of the minimum principal strain.

Q.3 **(a)** Draw and comment on the nature of the Mohr's circle diagram for the following cases where in the three principal stresses σ_1, σ_2 and σ_3 are given as: (i) unequal (ii) equal (iii) any two of them are equal. **07**

(b) A rubber cube is inserted in a cavity of the same form and size in a steel block and the top of the cube is pressed by a steel block with a pressure of "p" pascals. Considering the steel to be absolutely hard and assuming that there is no friction between steel and rubber, find (i) pressure of rubber against the box walls, and (ii) the extremum shear stresses in rubber. **07**

The Poison's ratio $\nu \leq 0.5$ and in usual notations

$\varepsilon_{xx} = [\sigma_x - \nu(\sigma_y + \sigma_z)]/E$

$\varepsilon_{yy} = [\sigma_y - \nu(\sigma_z + \sigma_x)]/E$

OR

Q.3 **(a)** Explain Menabrea's theorem. **07**

(b) Define modulus of rigidity, bulk modulus, and Poisson's ratio and show that for the bulk modulus to be positive, the value of Poisson's ratio cannot exceed a value 0.5. Also prove that materials with Poisson's ratio 0.5 are incompressible. **07**

Q.4 **(a)** Derive the Saint – venant's equations of compatibility. **07**

(b) A pipe made of steel has a tensile elastic limit $\sigma_y = 275$ MPa and $E = 207 \times 10^6$ kPa. If the pipe has an internal radius a = 5 cm and is subjected to an internal pressure $p = 70 \times 10^3$ kPa, determine the proper thickness for the pipe wall according to the major theories of failure. Use a factor of safety N = 4/3. **07**

OR

Q.4 **(a)** "Cubic dilatation is equal to the sum of three linear strains". Justify the statement with a mathematical proof. **07**

(b) A flat steel disk of 75 cm outside diameter with a 15 cm diameter hole is shrunk around a solid steel shaft. The shrink fit allowance is 1 part in 1000. $E = 2.18 \times 10^6$ kgf / cm^2. **07**

1) What are the stresses due to shrink fit?
2) At what rpm will the shrink fit loosen up as a result of rotation?
3) What is the circumferential stress in the disk when spinning at the above speed?

Assume that the same equations as for the disk are applicable to the solid rotating shaft also.

Q.5 **(a)** Explain uniform tension of an Anisotropic sheet with proper illustration. **07**

(b) "For a simply connected region, a steady temperature distribution with zero boundary traction will not affect the in-plane stress field". Justify the statement with a mathematical proof. **07**

OR

Q.5 **(a)** Give the displacement formulation of the anisotropic torsion problem. **07**

(b) Develop the compatibility equations for plane strain and plane stress for thermo elastic material. **07**

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