Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2013

Subject code: 710904 Date: 30-12-2013

Subject Name: Optimization Techniques

Time: 10.30 am – 01.00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Classify engineering optimization techniques with suitable examples based on a. Nature of constraints; b. Nature of equations involved ; c. Permissible values of design variables **07**

(b) A manufacturing firm produces two products, A and B, using two limited resources. The maximum amounts of resources 1 and 2 available per day are 1000 and 250 units, respectively. The production of 1 unit of product A requires 1 unit of resource 1 and 0.2 unit of resource 2, and the production of 1 unit of product B requires 0.5 unit of resource 1 and 0.5 unit of resource 2. The unit costs of resources 1 and 2 are given by the relations $(0.375 - 0.00005u_1)$ and $(0.75 - 0.0001u_2)$, respectively, where u_i denotes the number of units of resource i used (i = 1, 2). The selling prices per unit of products A and B, p_A and p_B, are given by **07**

$p_A = 2.00 - 0.0005x_A - 0.00015x_B$

$p_B = 3.50 - 0.0002x_A - 0.0015x_B$

where x_A and x_B indicate, respectively, the number of units of products A and B sold. Formulate the problem of maximizing the profit assuming that the firm can sell all the units it manufactures.

Q.2 **(a)** Obtain the dual problem of the following primal formulation. **07**

Maximize Z = 2X1 + 5X2 + 6X3; Subject to

5X1 + 6X2 -X3	<=	3
-2x1 + X2 + 4X3	<=	4
X1 - 5X2 + 3X3	<=	1
-3X1 - 3X2 + 7X3	<=	6
X1, X2, X3	>=	0

(b) Explain the significance of post optimality analysis of a simplex linear programming problem. **07**

How does simplex algorithm indicate that:

1. There is an alternate optimal solution?
2. The problem has unbounded optimal solution?
3. The problem has no feasible solution?

OR

(b) Determine the maximum and minimum values of the function **07**

$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$

Q.3 **(a)** Determine whether the constraint qualification and the Kuhn–Tucker conditions are satisfied at the optimum point. **07**

Minimize $f(x_1, x_2) = (x_1 - 1)^2 + x_2^2$

subject to

$g_1(x_1, x_2) = x_1^3 - 2x_2 \le 0$, $g_2(x_1, x_2) = x_1^3 + 2x_2 \le 0$

(b) Find the maximum of the function $f(\mathbf{X}) = 2x_1 + x_2 + 10$ subject to $g(\mathbf{X}) = x_1 + 2x_2^2 = 3$ using the Lagrange multiplier method. **07**

OR

Q.3 **(a)** Explain dynamic programming. How is it different from linear programming? Write four steps of dynamic programming. **07**

(b) Find the dimensions of a cylindrical tin (with top and bottom) made up of sheet metal to maximize its volume such that the total surface area is equal to $A_0 = 24\pi$. **07**

Q.4 **(a)** Solve the following LP problem using the branch-and-bound method: **07**
Maximize $f = 3x_1 + 4x_2$, subject to
$7x_1 + 11x_2 \leq 88$, $3x_1 - x_2 \leq 12$, $x_1 \geq 0$, $x_2 \geq 0$, x_i = integer, i = 1, 2

(b) A small machine tool manufacturing company entered into a contract to supply 80 drilling machines at the end of the first month and 120 at the end of the second month. The unit cost of manufacturing a drilling machine in any month is given by Rs.(50x + 0.2x2), where x denotes the number of drilling machines manufactured in that month. If the company manufactures more units than needed in the first month, there is an inventory carrying cost of Rs. 8 for each unit carried to the next month. Find the number of drilling machines to be manufactured in each month to minimize the total cost. Assume that the company has enough facilities to manufacture up to 200 drilling machines per month and that there is no initial inventory. Solve the problem as a final value problem. **07**

OR

Q.4 **(a)** In the context of dynamic programming explain the forward procedure and the backward procedure **07**

(b) Solve following LP model using cutting plane method **07**
Maximize z = 4x1 + 8x2
Subject to 4x1 + 5x2<= 40
x1 + 2x2<=12
x1, x2 >= 0; integers.

Q.5 **(a)** In the context of Linear Programming, explain the meaning and significance of sensitivity analysis with the help of suitable examples **07**

(b) Minimize $f(x,y) = k\, x^{-1} y^{-2}$ **07**
Subject to $g(x,y) = x^2 + y^2 - a^2 = 0$.
Find the solution using langrange multiplier method

OR

Q.5 **(a)** A firm has two factories X and Y and three retail stores A, B & C. The number of units of product available at factories X & Y are 200 & 300 respectively, while demanded at retail stores are 100, 150 & 250 resp. Rather than shipping directly from sources to destinations, it is decided to investigate the possibility of trans-shipment. Find the optimal shipping schedule. The transportation costs in rupees per unit are given below: **07**

	X	**Y**	**A**	**B**	**C**
X	0	6	7	8	9
Y	6	0	5	4	3
A	7	2	0	5	1
B	1	5	1	0	4
C	9	9	7	6	0

(b) Explain some of the practical application of integer programming problem **07**

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