Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2013

Subject code: 712306N **Date: 06-01-2014**

Subject Name: Quantum Computing Algorithm Analysis

Time: 10.30 am – 01.00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Suppose **07**

$$|\psi\rangle = 3|0\rangle - 2i|1\rangle, \quad |\phi\rangle = |0\rangle + 5|1\rangle$$

(i) Show that these states obey the Cauchy-Schwarz and triangle inequalities
(ii) Normalize the states.

(b) Explain divide and conquer strategy and also show how divide and conquer strategy can be applied to merge sort and quick sort with suitable example? **07**

Q.2 **(a)** Find the change of basis matrix to go from the computational basis $\{|0\rangle, |1\rangle\}$ to the $\{|\pm\rangle\}$ basis. Then use it to write **07**

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$$

and the operator $T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$ in the $\{|\pm\rangle\}$ basis.

(b) (i) Define surjection and injection with example. **07**
(ii) Show that the set of odd natural numbers is countable.

OR

(b) What is feasible solution? What is the difference between Greedy and Dynamic Programming? Solve Knapsack problem for W = <1, 3, 4, 5, 8, 9>, V = <1, 4, 7, 10, 11, 16> using Greedy Approach. Knapsack capacity is 23 units. Select items in descending order of profit. **07**

Q.3 **(a)** (i) If **05**

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \qquad find\ I \otimes Y|\psi\rangle.$$

(ii) Find the tensor product of the Pauli matrices X and Z. **02**

(b) Prove that the Huffman algorithm has a greedy choice property. What is an optimal Huffman code for the following set of frequencies, based on first 8 Fibonacci numbers? **07**

a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

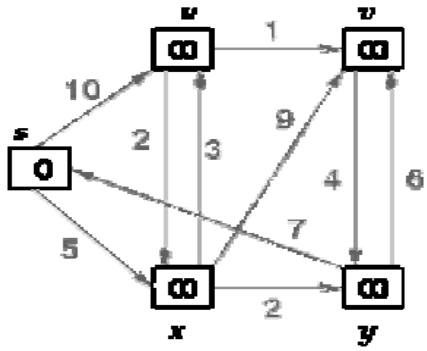
OR

Q.3 **(a)** A three-state system is in the state **07**

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|2\rangle$$

Write down the necessary projection operators and calculate the probabilities *Pr (0), Pr (1) and Pr (2).*

(b) Given a graph, find the shortest paths using Dijkstra's algorithm. **07**



Q.4 **(a)** Suppose **07**

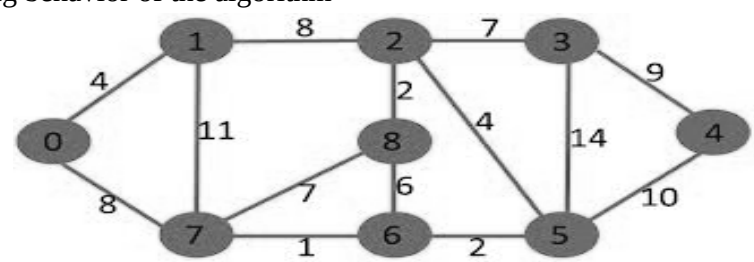
$$\rho = \begin{pmatrix} \frac{1}{3} & \frac{i}{3} \\ \frac{-i}{3} & \frac{2}{3} \end{pmatrix}$$

(a) Show that ρ is Hermitian and has positive eigenvalues that satisfy $0 \leq \lambda_i \leq 1$, and $\sum \lambda_i = 1$.

(b) Is this a mixed state?

(c) Find $\langle X \rangle$ for this state.

(b) Find minimum spanning tree using prim's algorithm for following graph, Comment on the running behavior of the algorithm **07**



OR

Q.4 **(a)** (i) Given that $\langle a|b\rangle = 4$ and $\langle c|d\rangle = 7$, calculate $\langle \psi|\phi\rangle$, where **04**

$$|\psi\rangle = |a\rangle \otimes |c\rangle \text{ and } |\phi\rangle = |b\rangle \otimes |d\rangle.$$

(ii) Calculate the tensor product of **03**

$$|a\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{and} \quad |b\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2} \\ 1 \end{pmatrix}$$

(b) Draw the undirected weighted graph. (1,2)=16,(1,6)=21,(1,5)=19,(2,3)=5, (2,4)=6,(2,6)=11,(3,4)=10,(3,2)=5, (4,6)=14,(4,5)=18,(5,6)=33,(5,4)=18. Find minimum spanning tree using kruskal's algorithm. **07**

Q.5 **(a)** Find the eigen values and eigenvectors for the "$\pi/8$"gate, which has the matrix representation **07**

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

(b) Find Longest Common Subsequence for given strings. **07**

A = XXYZWXYWX

B = YZWXYXY

OR

Q.5 **(a)** (i) Show that the trace of a matrix is equal to the sum of its eigen values for **04**

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 4 \\ 1 & 0 & 2 \end{pmatrix}$$

(ii) Explain the postulates of quantum mechanics. **03**

(b) Solve the following 0/1 knapsack problem using Dynamic Programming These are five items whose weights and values are given in following arrays. Weight w [] = {10, 15, 6, 9}, Profits p [] = {2, 5, 8, 1}. Show your equation and find out the optimal knapsack items for weight capacity of 11 units. **07**

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