GUJARAT TECHNOLOGICAL UNIVERSITY M. E. - SEMESTER – III • EXAMINATION – WINTER • 2013

Subje Subje	ect o ect N	code: 730403 Date: 28-11-2 Name: Optimization Techniques	013
Time	: 10	.30 am – 01.00 pm Total Marks	: 70
Instructions:			
	1. 2. 3.	Attempt all questions. Make suitable assumptions wherever necessary. Figures to the right indicate full marks.	
Q.1	(a)	(i) Find the maxima and minima, if any, of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 5.$	03
	(b)	(ii) Find the extreme points of the function $f(x_1, x_2) = x_1^3 + x_2^3 + x_1^2 + 2x_2^2 + 10.$ Find the dimensions of a restongular has of volume $K = 1000 \text{ im}^3$ for	04
	(0)	which the total length of the 12 edges is a minimum using the Lagrange's method of multipliers (No need to check the sufficiency conditions). Find the change in the dimensions of the box when the volume is changed to 1200 in^3 by using the value of λ^* found.	07
Q.2	(a)	Consider the following problem:	07
		Minimize $f(X) = (x_1 - 2)^2 + (x_2 - 1)^2$	
		subject to $2 \ge x_1 + x_2$, $x_2 \ge x_1^2$, Determine whether the Kuhn-Tucker conditions are satisfied at the following points:	
	(b)	$X_1 = [1.5, 0.5]^r$, $X_2 = [1, 1]^r$, $X_3 = [2, 0]^r$. Use the graphical method to minimize $f = -x_1 + 2x_2$ subject to the constraints	07
		$-x_1 + 3x_2 \le 10, x_1 + x_2 \le 6, x_1 - x_2 \le 2, \qquad x_1, x_2 \ge 0. OR$	
	(b)	Use Revised Simplex Method to maximize $f = 3x_1+2x_2+5x_3$ subject to $x_1 + 2x_2 + x_3 \le 430$, $3x_1 + 2x_3 \le 460$, $x_1 + 4x_2 \le 420$, $x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$.	07
Q.3	(a)	Use Fibonacci Method with $n = 6$ to maximize the function $f(\lambda) = \frac{0.5}{\sqrt{1 + \lambda^2}} - \sqrt{1 + \lambda^2} \left(1 - \frac{0.5}{1 + \lambda^2}\right) + \lambda$	07
	(b)	in the interval [0, 3]. Use Secant Method to find the minimum of $f = x^3 - 4x + 9$ with an initial step size of $t_0 = 0.2$, $\lambda_1 = 0.0$, and $\varepsilon = 0.01$.	07
Q.3	(a)	Use Dichotomous Search Method to find the minimum of $f = x^2 - 1.6x + 4$ in the interval (0.0, 1.00) to within 10% of the exact value. Take $\delta = 0.001$	07
	(b)	Use Newton's Method to minimize the function $\frac{1}{2}$	07
		$f(\lambda) = \frac{\lambda}{\log \lambda}$	
		with $\lambda_0 = 2$. Iterate until $ f'(\lambda) < 0.01$.	

Q.4 (a) Perform three iterations of Interval Halving Method to minimize the function $f(\lambda) = 3\lambda^5 - 5\lambda^3 - 30\lambda + 20$ in the interval (0, 2).

- **Q.5** (a) Minimize $f(x_1, x_2) = x_1 x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ from the starting point $X_1 = \begin{cases} 0 \\ 0 \end{cases}$ using Hooke and Jeeves's Method. Take $\varepsilon = 0.1$ and $\Delta x_1 = \Delta x_2 = 0.8$. Perform only two iterations.
 - (b) Find the dimensions of a rectangular prism type box that has the largest 07 volume when the sum of its length, width and height is limited to a maximum value of 60 *in*. and its length is restricted to a maximum value of 36 *in*.

OR

- **Q.5** (a) Use Fletcher-Reeves method to minimize $f(x_1, x_2) = 3x_1^2 - 5x_1x_2 + 4x_2^2 - 2x_1 + 3x_2$ from the point $X_1 = \begin{cases} 0 \\ 0 \end{cases}$. Perform only two iterations.
 - (b) Describe the basic approach of the penalty function method. Also write 07 the algorithm of Exterior Penalty function Method.

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