Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – II • EXAMINATION – WINTER • 2014

Subject code: 1720901 **Date: 02-12-2014**

Subject Name: Finite Element Method

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Differentiate between a LINK element and a BEAM element. **04**

(b) Figure 1 represents an assembly of three bar elements made of different materials. Determine the nodal displacements using penalty approach. Also find elemental stresses and support reactions. Assume that the displacement at node 4 is more than the gap of 0.5 mm, if support is not provided. Take P_2 = 50 kN. **10**

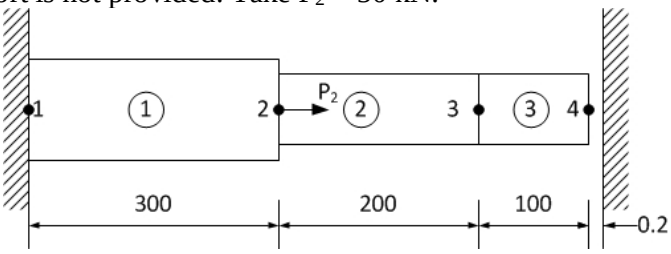

Figure 1

	Element 1	Element 2	Element 3
Material	Al	Al	Steel
Cross sectional area	800 mm^2	200 mm^2	200 mm^2
Modulus of Elasticity	70 GPa	70 GPa	200 GPa

Q.2 (a) Find the nodal displacement for the spring system shown in figure 2 using minimum potential energy principle. Take k_1 = 100 N/mm, k_2 = 150 N/mm, k_3 = 80 N/mm, k_4 = 120 N/mm, P_2 = 200 N and P_3 = 150 N. **07**

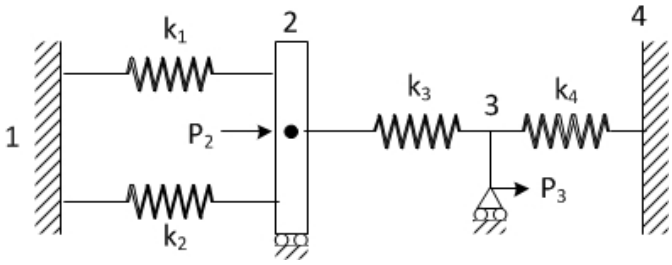

Figure 2

(b) A composite bar of 3 different materials, rigidly fixed at both the ends, is subjected to a uniform temperature rise of 80°C. In addition, axial load of 20 kN is applied at node 2 as shown in figure 3. Determine the nodal displacements. **07**

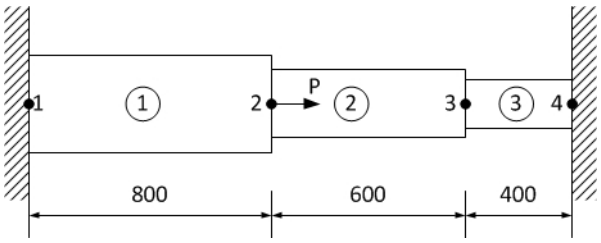

Figure 3

	Element 1	Element 2	Element 3
Material	Bronze	Aliminium	Steel
Cross sectional area	2400 mm^2	1200 mm^2	600 mm^2
Modulus of elasticity	83 GPa	70 GPa	200 GPa
Coeff. of Thermal Exp.	18.9×10^{-6}	23×10^{-6}	11.7×10^{-6}

OR

(b) Determine the nodal displacements, element stresses and support reactions in the bar axially load as shown in figure 4. **07**

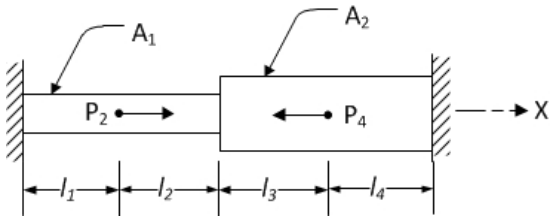

Figure 4

$A_1 = 25\ mm^2$, $A_2 = 50\ mm^2$, $l_1 = l_2 = l_3 = l_4 = 50$ mm, $P_2 = 25$ N, $P_4 = 20$ N and $E = 70 \times 10^3\ N/mm^2$

Q.3 (a) With the help of illustrative examples, explain the terms: (i) Plane strain and (ii) Plane strain. Write the stress-strain relationship for both plane stress and plane stress conditions. **04**

(b) Find the displacements at nodes, stresses in members, and reactions at supports for the truss shown in figure 5. The Young's modulus $E = 20 \times 10^6\ N/cm^2$, Cross sectional area of element 1 and 3= 6 cm^2 and of element 2 = 5 cm^2. **10**

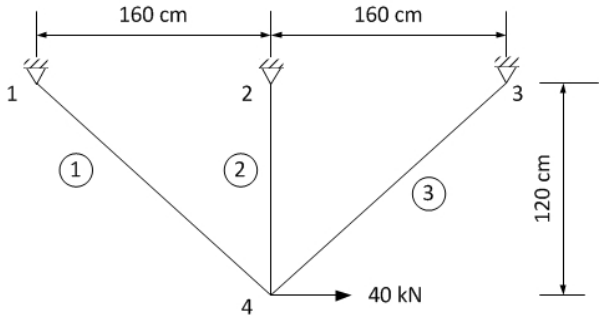

Figure 5

OR

Q.3 (a) What is meant by axisymmetric solid? State the conditions to be satisfied in order to use axisymmetric elements. **04**

(b) For the truss shown in Figure 6, use symmetry to determine the displacements of the nodes and the stresses in each element. All elements have $E = 200 \times 10^3\ N/mm^2$. Elements 1, 2, 3, and 5 have $A = 4000\ mm^2$ and element 4 has $A = 8000\ mm^2$. **10**

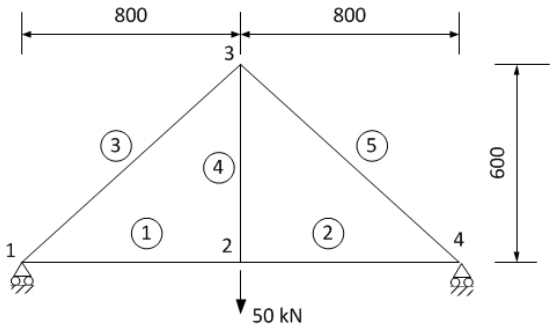

Figure 6

Q.4 (a) Explain the difference between a beam element and a frame element. Using the direct stiffness matrix approach and the stiffness matrices for both bar and beam elements **04**

known, derive the stiffness matrix for a planar frame element.

(b) For the beam shown in figure 7, calculate the nodal displacement and reactions. Given: E = 200 GPA, I = 2 x 10^{-4} m^4. **10**

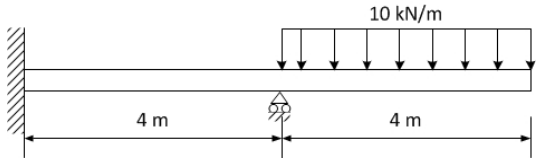

Figure 7

OR

Q.4 (a) The element shown in the figure 8 is from a finite element model of a thin aluminum plate with 5 mm thickness. The nodal displacements vector q = [0.05 0.02 0.04 0.04 0.03 0.03]^T mm. Determine: (a) the natural coordinates (,) of point P, and (b) the displacement at the point P. **04**

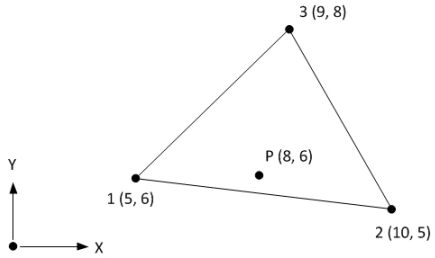

Figure 8

(b) For the beam shown in figure 9, calculate the nodal displacement. **10**

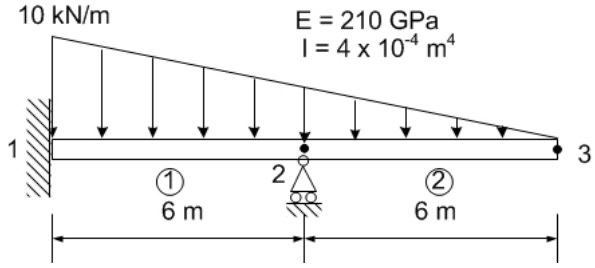

Figure 9

Q.5 (a) What do you understand by lower order and higher order elements? Give examples of higher and lower order elements for 1D, 2D and 3D cases. **07**

(b) Find the angle of twist at nodes 2 and 3 for the shaft subjected to twisting moments as shown in figure 10. Take: D_1 = 50 mm, D_2 = 25 mm, T_2 = 100 N-mm, T_3 = 50 N-mm and G = 60 GPa. **07**

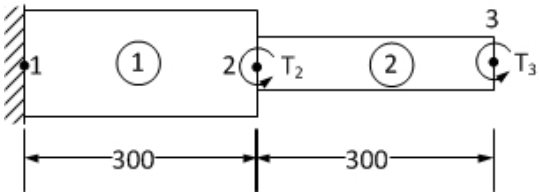

Figure 10

OR

Q.5 (a) Derive the shape functions in terms of natural coordinate system for a four noded quadrilateral element using inspection method. Also draw the variation in shape functions. **07**

(b) A mild steel rod of 20 mm diameter and 300 mm length is enclosed inside a hollow copper tube of external diameter 30 mm and internal diameter 20 mm. The ends of the rod and tube are braced together and the composite bar is subjected to an axial pull of 40 kN. If E for steel is 200 GPa and that for copper is 70 GPa find the extension of the composite bar and the stresses developed in each. **07**

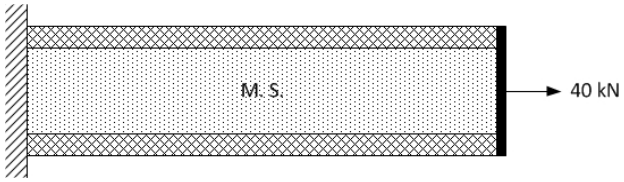

Figure 11