Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – II • EXAMINATION – WINTER • 2014

Subject code: 1722309 **Date: 08-12-2014**

Subject Name: Numerical Methods

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Write algorithm of Newton Raphson Method. Also, explain its pitfalls. **07**

(b) Use Bisection method to determine drag coefficient c for a parachutist after free falling, using the nonlinear equation $f(c) = \left(\frac{667.38}{c}\right)\left(1 - e^{-(0.146843c)}\right) - 40 = 0$. Taking initial guesses (14.5 15) and stopping criteria 0.5%. **07**

Q.2 **(a)** Give geometrical interpretation of False Position method and use it to find the real root of $f(x) = xe^x - 2 = 0$ which lies between 0.8 and 0.9 correct to three decimal places. **07**

(b) What is mathematical modeling? Develop a model to find the velocity v of a freely falling parachutist as a function of time assuming that the air resistance is linearly proportional to v, How can you solve the problem numerically? **07**

OR

(b) Use following table to compute value of the Bessel function $J_o(x)$ for **07**
1) x=1.72 using Newtonôs forward difference interpolation formula and
2) x=1.93 using Newtonôs backward difference interpolation formula.

x	1.7	1.8	1.9	2.0
f(x)	0.397 9849	0.339 9864	0.281 8186	0.223 8908

Q.3 **(a)** Describe Algorithm for the Gauss- Seidel Method. **07**

(b) 1) What is partial pivoting? Why do we use it? **07**

2) Use Gauss Elimination method to solve the following system of equations.

$$3x + 2y + z = 10$$
$$2x + 3y + 2z = 14$$
$$x + 2y + 3z = 14$$

OR

Q.3 **(a)** For a spring mass system, following equations are obtained. Use Gauss Jordan method to find displacements x_1, x_2, x_3 **07**

$$30x_1 - 20x_2 = 19.6$$
$$-20x_1 + 30x_2 - 10x_3 = 29.4$$
$$-10x_2 + 10x_3 = 24.5$$

(b) The following system of equations was generated by applying Kirchhoffôs law to an electric circuit. Use Gauss Seidel method upto three iterations to determine the currents I_1, I_2 *and* I_3. **07**

$$3I_1 - 0.1I_2 - 0.2I_3 = 7.85$$
$$0.1I_1 + 7I_2 - 0.3I_3 = -19.3$$
$$0.3I_1 - 0.2I_2 + 10I_3 = 71.4$$

Q.4 **(a)** Derive normal equations for polynomial regression to fit a second degree polynomial for the given data points $(x_i, y_i), i = 1,2,...n$ using least square method. **07**

(b) Fit first order Spline to the data given in the following table. Also, evaluate the function at $x = 5$. **07**

x	3	4.5	7	9
$f(x)$	2.5	1.0	2.5	0.5

OR

Q.4 **(a)** What is difference between the two methods of curve fitting, namely Regression and interpolation? **07**

The voltage v across a capacitor at time t second is given by the following table: Use the method of least square to fit a curve $v = ae^{kt}$ to this data.

t	0	2	4	6	8
v	150	63	28	12	5.6

(b) The following data was taken from an experiment. Use linear regression to fit a straight line for data given in the following table. Also, find the standard error of the estimate. **07**

X	1	2	3	4	5	6	7
Y	0.5	2.5	2.0	4.0	3.5	6.0	5.5

Q.5 **(a)** 1) Why do we need methods of Numerical Integration? **02**

2)A wind force distributed against the side of a skyscraper is measured as **05**

Height L (m)	0	30	60	90	120	150	180	210	240
Force F(N/m)	0	350	1000	1500	2600	3000	3300	3500	3600

Calculate the net force using Simpson's 1/3 rule.

(b) Use Runge Kutta forth order method to calculate y(0.2) given $\frac{dy}{dx} = x + y,\ y(0) = 1$, taking h=0.1. **07**

OR

Q.5 **(a)** Use Heun's method to integrate $y' = \frac{2y}{x}$ $y(1) = 2$ to estimate $y(1.5)$, taking step size h=0.25 and only one iteration in the corrector step. **07**

(b) A steady state heat balance for a 10-m rod can be represented as **07**

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0 \text{ with } h' = 0.01\text{m}^{-2},\ T_a = 20^oC,\ T(0) = 40 \text{ and } T(10) = 200$$

Use the finite difference approach with $\Delta x = 2\,\text{m}$ to solve the boundary value problem.