

GUJARAT TECHNOLOGICAL UNIVERSITY**ME - SEMESTER- I • EXAMINATION – WINTER 2014****Subject Code: 2712001****Date: 07/01/ 2015****Subject Name: Matrix Methods of Structural Analysis****Time: 02:30 p.m. to 05:00 p.m.****Total Marks: 70****Instructions:**

1. Take $E = 2 \times 10^8 \text{ kN/m}^2$, $I_Z = I_Y = 0.001 \text{ m}^4$, $A_X = 0.01 \text{ m}^2$, $I_X = 0.002 \text{ m}^4$ and $G = 8 \times 10^7 \text{ kN/m}^2$ unless and otherwise given.
2. Attempt all questions.
3. Make suitable assumptions wherever necessary and mentioned it clearly.
4. Figures to the right indicate full marks.

Q.1 (a) Derive the relation $S_{MS} = R_T^T S_M R_T$ with usual notations. Also write R_T matrix for plane truss, plane frame and grid structures. **07**

(b) Obtain the rearranged joint stiffness matrix for the continuous beam shown in Figure 1. Flexural rigidity of all members is constant. **07**

Q.2 (a) Obtain the rearranged joint stiffness matrix for the grid shown in Figure 2. Flexural and torsional rigidity of all members are constant. **07**

(b) Determine free joint displacements and support reactions for the grid structure shown in Figure 2 using stiffness member approach. **07**

OR

(b) Obtain load vector for the continuous beam shown in Figure 1, considering roller supports at B and C in place of elastic supports, if (i) temperature of AB member is increased so that the top and bottom fibers are at 45°C and 55°C , respectively (ii) support A rotates counterclockwise by 0.02 radian and (iii) support C sinks by 5 mm. Assume depth of members as 300 mm and coefficient of thermal expansion as 1.2×10^{-6} per $^\circ\text{C}$. **07**

Q.3 (a) For the truss shown in Figure 3, obtain rearranged joint stiffness matrix taking advantage of symmetry. **07**

(b) Determine free joint displacements and support reactions for the truss shown in Figure 3 using stiffness member approach. **07**

OR

Q.3 (a) Construct the rearranged joint stiffness matrix for the plane frame shown in Figure 4. **07**

(b) Determine free joint displacements and support reactions for the plane frame shown in Figure 4 using stiffness member approach. **07**

Q.4 (a) Obtain assembled flexibility matrix for the truss shown in Figure 5. Cross sectional area of all members is A. **07**

(b) Calculate member end-actions for the truss shown in Figure 5 using flexibility member approach. Tabulate the member forces. **07**

OR

Q.4 (a) Obtain assembled flexibility matrix for the plane frame shown in Figure 6. Flexural rigidity of all members is constant. **07**

Q.4 (b) Calculate support reactions for the plane frame shown in Figure 6 using flexibility member approach. **07**

- Q.5 (a)** Obtain the rearranged joint stiffness matrix of a composite structure shown in Figure 7. AB is a beam member with cross-section of 230 mm x 230 mm while BC is a truss member with 50 mm cross sectional diameter. Modulus of elasticity of beam and truss member is 20GPa and 200 GPa, respectively. **07**
- (b)** Determine free joint displacements and support reactions for the composite structure shown in Figure 7 using stiffness member approach. **07**
- OR**
- Q.5 (a)** Derive rotation transformation matrix for a space truss member. **07**
- (b)** Define the types of non-linearity, give the assumptions of non-linear analysis and list the methods of non-linear analysis **07**

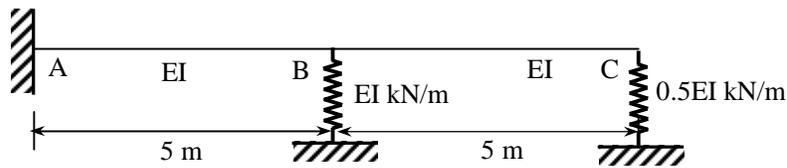


Figure 1

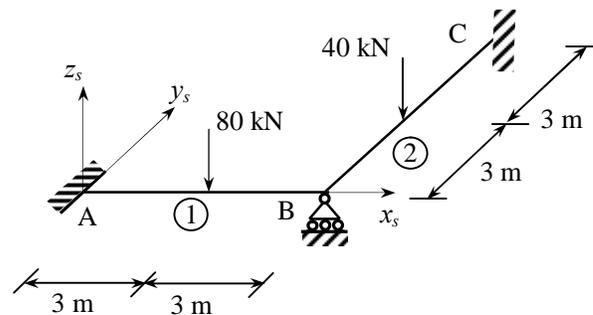


Figure 2

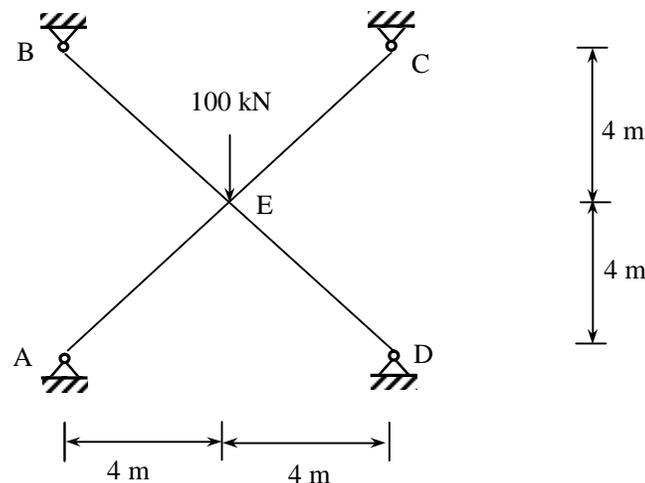


Figure 3

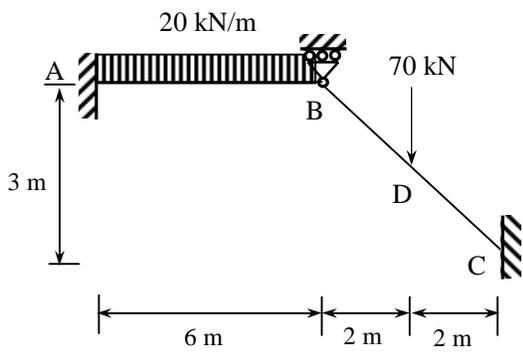


Figure 4

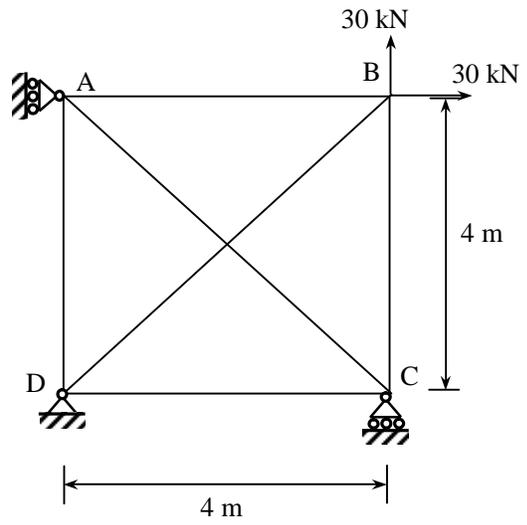


Figure 5

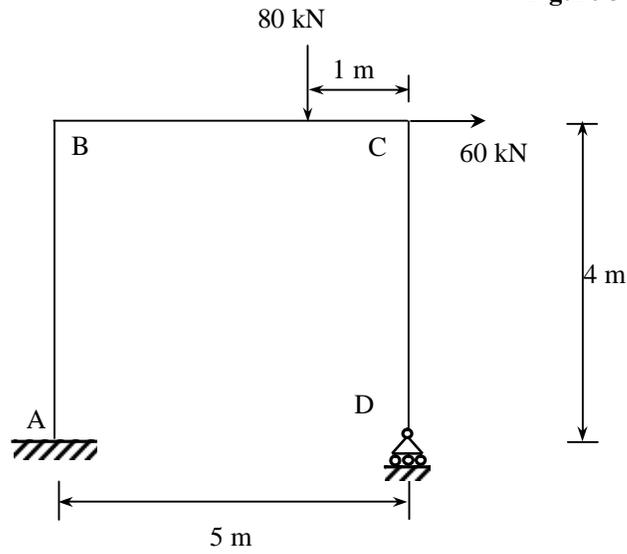


Figure 6

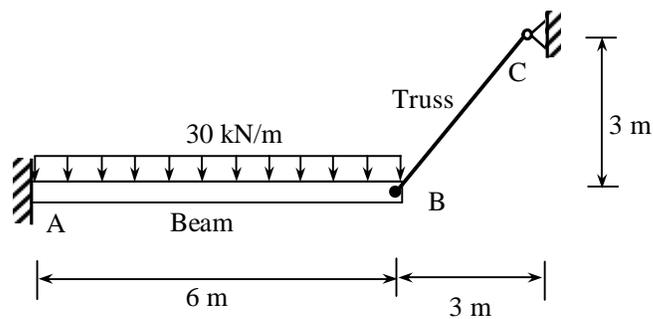


Figure 7

Plane Frame Member Stiffness Matrix for Structure Axes

$$S_{MSi} = \begin{bmatrix} \frac{EA_X}{L} C_X^2 + \frac{12EI_Z}{L^3} C_Y^2 & \left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & -\frac{6EI_Z}{L^2} C_Y & -\left(\frac{EA_X}{L} C_X^2 + \frac{12EI_Z}{L^3} C_Y^2\right) & -\left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & -\frac{6EI_Z}{L^2} C_Y \\ \left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & \frac{EA_X}{L} C_Y^2 + \frac{12EI_Z}{L^3} C_X^2 & \frac{6EI_Z}{L^2} C_X & -\left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & -\left(\frac{EA_X}{L} C_Y^2 + \frac{12EI_Z}{L^3} C_X^2\right) & \frac{6EI_Z}{L^2} C_X \\ -\frac{6EI_Z}{L^2} C_Y & \frac{6EI_Z}{L^2} C_X & \frac{4EI_Z}{L} & \frac{6EI_Z}{L^2} C_Y & -\frac{6EI_Z}{L^2} C_X & \frac{2EI_Z}{L} \\ -\left(\frac{EA_X}{L} C_X^2 + \frac{12EI_Z}{L^3} C_Y^2\right) & -\left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & \frac{6EI_Z}{L^2} C_Y & \frac{EA_X}{L} C_X^2 + \frac{12EI_Z}{L^3} C_Y^2 & \left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & \frac{6EI_Z}{L^2} C_Y \\ -\left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & -\left(\frac{EA_X}{L} C_Y^2 + \frac{12EI_Z}{L^3} C_X^2\right) & -\frac{6EI_Z}{L^2} C_X & \left(\frac{EA_X}{L} - \frac{12EI_Z}{L^3}\right) C_X C_Y & \left(\frac{EA_X}{L} C_Y^2 + \frac{12EI_Z}{L^3} C_X^2\right) & -\frac{6EI_Z}{L^2} C_X \\ -\frac{6EI_Z}{L^2} C_Y & \frac{6EI_Z}{L^2} C_X & \frac{2EI_Z}{L} & \frac{6EI_Z}{L^2} C_Y & -\frac{6EI_Z}{L^2} C_X & \frac{4EI_Z}{L} \end{bmatrix}$$

Grid Member Stiffness Matrix for Structure Axes

$$S_{MSi} = \begin{bmatrix} \frac{GI_X}{L} C_X^2 + \frac{4EI_Y}{L} C_Y^2 & \left(\frac{GI_X}{L} - \frac{4EI_Y}{L}\right) C_X C_Y & \frac{6EI_Y}{L^2} C_Y & -\frac{GI_X}{L} C_X^2 + \frac{2EI_Y}{L} C_Y^2 & -\left(\frac{GI_X}{L} + \frac{2EI_Y}{L}\right) C_X C_Y & -\frac{6EI_Y}{L^2} C_Y \\ \left(\frac{GI_X}{L} - \frac{4EI_Y}{L}\right) C_X C_Y & \frac{GI_X}{L} C_Y^2 + \frac{4EI_Y}{L} C_X^2 & -\frac{6EI_Y}{L^2} C_X & -\left(\frac{GI_X}{L} + \frac{2EI_Y}{L}\right) C_X C_Y & -\frac{GI_X}{L} C_Y^2 + \frac{2EI_Y}{L} C_X^2 & \frac{6EI_Y}{L^2} C_X \\ \frac{6EI_Y}{L^2} C_Y & -\frac{6EI_Y}{L^2} C_X & \frac{12EI_Y}{L^3} & \frac{6EI_Y}{L^2} C_Y & -\frac{6EI_Y}{L^2} C_X & -\frac{12EI_Y}{L^3} \\ -\frac{GI_X}{L} C_X^2 + \frac{2EI_Y}{L} C_Y^2 & -\left(\frac{GI_X}{L} + \frac{2EI_Y}{L}\right) C_X C_Y & \frac{6EI_Y}{L^2} C_Y & \frac{GI_X}{L} C_X^2 + \frac{4EI_Y}{L} C_Y^2 & \left(\frac{GI_X}{L} - \frac{4EI_Y}{L}\right) C_X C_Y & -\frac{6EI_Y}{L^2} C_Y \\ -\left(\frac{GI_X}{L} + \frac{2EI_Y}{L}\right) C_X C_Y & -\frac{GI_X}{L} C_Y^2 + \frac{2EI_Y}{L} C_X^2 & -\frac{6EI_Y}{L^2} C_X & \left(\frac{GI_X}{L} - \frac{4EI_Y}{L}\right) C_X C_Y & \frac{GI_X}{L} C_Y^2 + \frac{4EI_Y}{L} C_X^2 & \frac{6EI_Y}{L^2} C_X \\ -\frac{6EI_Y}{L^2} C_Y & \frac{6EI_Y}{L^2} C_X & -\frac{12EI_Y}{L^3} & -\frac{6EI_Y}{L^2} C_Y & \frac{6EI_Y}{L^2} C_X & \frac{12EI_Y}{L^3} \end{bmatrix}$$