## GUJARAT TECHNOLOGICAL UNIVERSITY ME - SEMESTER-1 • EXAMINATION – WINTER 2014

Subject Code: 2714303	Date:09/01/ 2015
Subject Name: Theory of Elasticity & Plasticity	
Гіте: 2:30 to 5:00 Рт	Total Marks: 70
Instructions:	
1. Attempt all questions.	
2. Make suitable assumptions wherever necessary.	

3. Figures to the right indicate full marks

- Q.1 (a) Derive the basic differential equation for equilibrium in Cartesian co- 07 ordinate system.
  - (b) If the components of the stress tensor in MPa, at some point P in a beam 07 with respect to (x,y,z) co-ordinates is given as below:

- (a) Find the stress vector  $t_n$  on the plane whose normal is given by nx=ny=nz =  $1/\sqrt{3}$
- (b) Find out the normal (n) and shear (n) components of  $t_n$ .

Q.2 (a) Commencing from the equations defining the state of stress at a point derive 07 the general stress relationship for the normal stress on an inclined plane.  $\sigma_n = \sigma_{xx}l^2 + \sigma_{zz}n^2 + \sigma_{yy}m^2 + 2\sigma_{xy}lm + 2\sigma_{yz}mn + 2\sigma_{zx}ln$ 

Show that the relationship reduces for the plane stress system to well-known equation:

$$\sigma_n = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \frac{1}{2}(\sigma_{xx} - \sigma_{yy})\cos 2\theta + \sigma_{xy}\sin 2\theta$$

(b) Explain Compatibility of strain in detail. Derive the compatibility relations 07 of strain for a 2-D system.

OR

- (b) Draw a neat sketch of the displacement of a cubic element and hence derive 07 the equations of linear and shear strains in Cartesian co-ordinate system.
- Q.3 (a) Derive Airyø stress function in Polar co-ordinate system for an axi- 07 symmetric stress distribution.
  - (b) The stress tensor at a point is given as  $\begin{bmatrix} 200 & 160 & -120 \\ 160 & -240 & 100 \\ -120 & 100 & 160 \end{bmatrix} kN/m^2$  07

Determine the strain tensor at this point. Take  $E=210 \times 10^6 \text{ kN/m}^2$  and  $\mu=0.3$ 

## OR

Q.3 (a) For the curved beam subjected to moment: M = 150 kJ, internal & external 07 radii: a = 150 mm & b = 350 mm respectively, calculate radial and transverse stresses at inner, outer and every quarter thickness points and plot their variations using the following equations with usual notations:

 $\begin{array}{l} \mbox{Radial stress:} \\ \sigma_r &= \acute{0} \ ^{4M}\!/_N \left[ a^2 b^2 / r^2 \, ln \, (b/a) + b^2 \, ln \, (r/b) + a^2 \, ln \, (a/r) \right] \\ \mbox{Tangential stress:} \\ \sigma_\theta &= \acute{0} \ ^{4M}\!/_N \left[ \acute{0} \ a^2 b^2 / r^2 \, ln \, (b/a) + b^2 \, ln \, (r/b) + a^2 \, ln \, (a/r) + b^2 \, \acute{0} \, a^2 \right] \\ \mbox{Here; } N &= (b^2 \ \acute{0} \ a^2)^2 \, \acute{0} \ 4 \ a^2 b^2 \, \left[ ln \, (b/a) \right]^2 \\ \end{array}$ 

- (b) Using Swift construction, find normal and resultant shear stress on a plane 07 whose normal has directions cosines are l = 0.342, m = 0.405 respectively w.r.t. Principal stresses:  $P_1 = 900$  MPa (Tensile),  $P_2 = 200$  MPa (Tensile) and  $P_3 = 300$  MPa (Compressive).
- Q.4 (a) State the generalized Hookeøs law. Write down the constitutive relations for 10 an Isotropic material in 2D. Calculate the volumetric strain for the following data:  $x = 200 \text{ N/mm}^2$ ,  $y = 150 \text{ N/mm}^2$ ,  $z = 120 \text{ N/mm}^2$ ,  $E = 210 \text{ kN/mm}^2$ ,  $\mu = 0.3$ .
  - (b) State the various failure criterion that govern the failure of materials under 04 static loading. Also give the statement of Tresca and Von Mises yield criterion.

## OR

Q.4 (a) If the stress tensor for a three dimensional stress system is given as below 07 and one of the principal stress has a value of 40  $MN/m^2$ , determine the

	30	10	10	
values of the three eigen vectors.	10	0	20	
	10	<b>20</b>	0	

Q.4 (b) Prove the following failure criteria according to the Distortion Energy 07 theory:

$$2\sigma_y^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

- Q.5 (a) Explain the concept of stability of structures. Give the basis of stability of 07 analysis for a slender straight column as well as for a column initially bent.
  - (b) Draw the stress distribution diagram for thick wall cylinder subjected to only 07 internal pressure. Also explain the boundary conditions utilized in TOE for
    - (i) Thick wall cylinder subjected to only external pressure
    - (ii) Plate element with circular hole of large radius

## OR

- Q.5 (a) Draw a neat sketch of an element subjected to body forces, radial stresses, 07 transverse stresses and shear stresses, derive the basic differential equations of equilibrium in 2-D Polar co-ordinate system.
  - (b) Write Short notes on any two:

07

- (i) Parameters that govern the theories of failure
- (ii) Assumptions made in Linear / Classical theory of elasticity
- (iii)Hydrostatic and Deviatoric stresses

(iv)Ductile failure vs Brittle failure