Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 710107N **Date: 03-12-2014**

Subject Name: Quantum Theory and Algorithm Design

Time: 10:30 am - 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Write the definition of Basis and Dimension of a vector space. Prove that a spanning set of basis vectors for a given vector space V is not unique. **07**

(b) Describe the Orthonormality in vector space. Suppose that $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$ is an orthonormal basis for a three dimensional Hilbert Space. A system is in the state $|\psi\rangle = \frac{1}{\sqrt{5}}|u_1\rangle - i\sqrt{\frac{7}{15}}|u_2\rangle + \frac{1}{\sqrt{3}}|u_3\rangle$ **07**

i. Is this state normalized?

ii. If a measurement is made, find the probability of finding the system in each of the states $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$

Q.2 **(a)** What is the procedure for obtaining the Eigen value λ and Eigen vector $|\psi\rangle$ of an operator A? Find the Eigen values and Eigen vectors for the $\frac{\pi}{8}$ gate, which has the matrix representation $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix}$ **07**

(b) i. Write Primôs Algorithm. **03**

ii. Explain Heisenberg uncertainty principle. **04**

OR

(b) i. Write the Bellman Ford algorithm. **04**

ii. Show that $\{\sigma_i, \sigma_j\} = 0$ when $i \neq j$. Where σ_i and σ_j are the Pauli operators. **03**

Q.3 **(a)** What is the Trace of an operator? If an operator A expressed in $\{|0\rangle, |1\rangle\}$ basis is given by $A = 2i|0\rangle\langle 0| + 3|0\rangle\langle 1| - 2|1\rangle\langle 0| + 4|1\rangle\langle 1|$ then find the Trace. **07**

(b) Prove the following statements involving trace operations. **07**

i. The trace is cyclic i.e. $T_r(ABC) = T_r(CAB) = T_r(BCA)$

ii. The trace of an operator is equal to the sum of its Eigen values. If the Eigen values of A are labeled as λ_i then $T_r(A) = \sum_{i=1}^{n} \lambda_i$

iii. $T_r(A|\phi\rangle\langle\psi|) = \langle\psi|A|\phi\rangle$

OR

Q.3 **(a)** What is the expectation value of an operator? An operator A acts on the qutrit basis states in the following way **07**

$A|0\rangle = |1\rangle$, $A|1\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, $A|2\rangle = 0$

Find the expectation value of operator A, i.e., $\langle A\rangle$ for the state $|\psi\rangle = \frac{1}{2}|0\rangle - \frac{i}{2}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$

(b) Explain divide and conquer methodology with its application on merge sort, in detail. **07**

Q.4 **(a)** What is the Projection operator? A three state system is in the state $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|2\rangle$ write down the necessary projection operators and calculate the probabilities $P_\psi(0), P_\psi(1), P_\psi(2)$ **07**

(b) Write all the Postulates of Quantum Mechanics. **07**

OR

Q.4 **(a)** Suppose that in a 0-1 knapsack problem, the order of the items when sorted by increasing weight is the same as their order when sorted by decreasing value. Give an efficient algorithm to find an optimal solution to this variant of the knapsack problem, and also discuss that your algorithm is correct. **07**

Q.4 **(b)** Explain how an operator acts on tensor products. Suppose $|\psi\rangle = |a\rangle \otimes |b\rangle$ and $A|a\rangle = a|a\rangle$, $B|b\rangle = b|b\rangle$ then what is $A \otimes B|\psi\rangle$? **07**

Q.5 **(a)** Suppose that instead of always selecting the first activity to finish, we instead select the last activity to start that is compatible with all previously selected activities. Describe how this approach is a greedy algorithm and prove that it yields an optimal solution. **07**

(b) Explain the properties which are satisfied by the tensor product of two operators $A \otimes B$. Suppose that A is a projection operator in H_1 where $A = |0\rangle\langle 0|$ and B is projection operator in H_2 where $B = |1\rangle\langle 1|$. Find $A \otimes B|\psi\rangle$ where $|\psi\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$ **07**

OR

Q.5 **(a)** What is density operator? A system is in the state, defined as $|\psi\rangle = \frac{1}{\sqrt{3}}|u_1\rangle + i\sqrt{\frac{2}{3}}|u_2\rangle$, where the $|u_k\rangle$ constitute an orthonormal basis. Write down the density operator and show that it has unit trace. **07**

(b) Write the algorithms for the operations of Insertion and deletion on Binary Search Tree. **07**

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