Seat No.: _____ *Enrolment No._____*

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 710401N **Date: 01-12-2014**

Subject Name: Statistical Signal Analysis

Time: 10:30 am - 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** What is significance of Cumulative Distribution Function (CDF). Show that Cumulative Distribution function (CDF) of random variable is non-decreasing function. **07**

(b) The step CDF is given as, **07**

$F_X(x)=0.24\ U(x-x_a) + 0.32\ U(x-x_b) + 0.44\ U(x-x_c)$

Where U(.) is the unit step function. Find the PDF of $F_X(x)$ and plot it.

Q.2 **(a)** The PDF of a Gaussian variable X is given as $P_x(x) = \frac{1}{3\sqrt{2\pi}} e^{\frac{-(x-4)^2}{18}}$ **07**

Determine : (a) $P(x \geq 4)$ (b) $P(x \geq 0)$ (c) $P(x \geq -2)$

(b) The joint Probability Density Function (PDF) of two random variables X and Y is given as **07**

$P_x(x)= C(2x+y)$ for $0 \leq x \leq 2,\ 0 \leq y \leq 3$

$= 0$ otherwise

Determine the value of constant C

OR

(b) A uniformly distributed Random variable X has Probability Density Function (PDF) $f_X(x) = 1/2\pi,\ 0 \leq X \leq 2\pi$. Find E [cos(x)] and $E[x^2]$. **07**

Q.3 **(a)** Explain the central limit theorem with proper example. **07**

(b) Let Φ be a uniformly distributed random variable in the interval $-\pi/2 \leq \Phi \leq \pi/2$. Suppose random variable X is defined as, **07**

$X=a \tan[\Phi]$, What will be the distribution of X?

OR

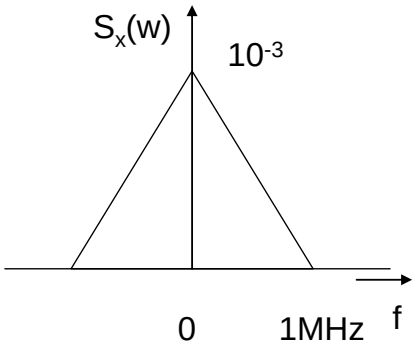
Q.3 **(a)** What is Chebyshave's inequality? Where it is used? Write short note on it. **07**

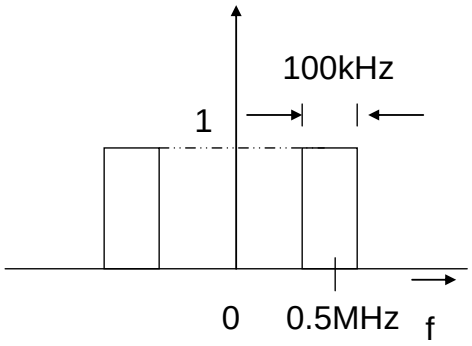
(b) If one realization of random process has following data. Plot PSD ,G(f) and auto correlation function $R(\tau)$ of the process, **07**

Data:

(1) Signal has DC value of 2 volts. (2) RMS value is 4 Volts. (3) Vr1(t) and Vr2(t+ τ) are independent for $|\tau| \leq 5$ μsec. (4) $R(\tau)$ decreases linearly with $|\tau|$ for $0 \leq \tau \leq 5$ μsec.

Q.4 **(a)** A random process x(t) with the PSD shown in figure below is passed through a band pass filter. Determine the PSD and mean square value of the Quadrature component of the output process. Center frequency of band pass filter is 0.5 MHz. **07**





(b) With proper sketch show Sample space, Sample function, Random process and random variable for time varying process. **07**

Sketch the ensemble of the random Process $X(t)=\mu t+\lambda$ under following two cases.

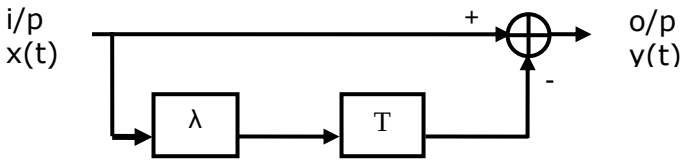
Case 1 :Where λ is a constant and μ is an Random variable uniformly distributed in the range (-2,2).

Case 2 : where μ is a constant and λ is a Random variable uniformly distributed in the range (-2,2).

State whether this is stationary or non stationary random process under both the cases 1 and 2.

OR

Q.4 **(a)** The autocorrelation function of a stochastic process X(t) is $Rx(\tau)=0.5No\delta(\tau)$. Such process is called white process. If X(t) is the input to system as shown below find the power spectral density at output of the system. **07**



-Attenuation is λ

-Delay is τ

(b) What is a random process? Give classification of random processes with proper examples. **07**

Q.5 **(a)** In linear estimation of X from Y, show that error in estimation is always orthogonal any linear OR nonlinear function of Y. **07**

[Note: prove this result for any density function]

(b) Define Almost sure convergence, Mean square sense convergence. **07**

OR

Q.5 **(a)** As an expert in SSA, one curious person ask you whether the functions sin(4t) and $\delta(t-2)$ are the valid covariance functions of real stationary process or not. Give answer of gentleman's query with proper justification. **07**

(b) State the significance of convergence and write short note on convergence in probability. **07**

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