Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 710901N Date: 01-12-2014

Subject Name: Theory of Elasticity

Time: 10:30 am - 01:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	Draw and comment on the nature of the Mohr's circle diagram for the following cases where in the three principal stresses σ_1, σ_2 and σ_3 are given as: (i) unequal (ii) equal (iii) any two of them are equal	**07**
	(b)	Following state of strain exist at point a P; $\varepsilon_{xx} = 0.02; \varepsilon_{yy} = 0.06; \varepsilon_{zz} = 0; \gamma_{xy} = -0.04; \gamma_{yz} = -0.02; \gamma_{zx} = 0$ Determine the principal strains and the direction of the maximum and minimum principal strains.	**07**
Q.2	**(a)**	"Cubical dilatation is equal to sum of three linear strains", justify the statement with the help of mathematical derivation.	**07**
	(b)	The state of stress at a point is characterized by the components, σ_x = 100 MPa, σ_y = -40 MPa, σ_z = 80 MPa, $\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$. Determine the extremum values of the shear stresses, their associated normal stresses, the octahedral shear stress and its associated normal stress.	**07**
		OR	
	(b)	At point P in a body, σ_x = 10,000 N/cm^2, σ_y = -5,000 N/cm^2, σ_z = -5,000 N/cm^2, $\tau_{xy} = \tau_{yz} = \tau_{zx}$ = 10,000 N/cm^2. Determine the normal and shearing stresses on a plane that is equally inclined to all the three axes.	**07**
Q.3	**(a)**	For the plane state of stress conditions determine the values of two principal stresses and maximum shear stress.	**07**
	(b)	(i). Define bulk modulus and derive the equation $K = (1/3)(3\lambda+2\mu)$ where, K = bulk modulus, λ and μ are Lame's constants.	**04**
		(ii). Compute Lame's constants λ and μ for steel having E = 207 x 10^6 kPa and $\nu = 0.3$.	**03**
		OR	
Q.3	**(a)**	Derive Saint-Venant's equations of compatibility.	**07**
	(b)	(i). Write Generalized Hook's law for the homogeneous linearly elastic material.	**03**
		(ii). "There are only two elastic constants involved in relation between the principal stresses and principal strains for isotropic materials". Justify the statement based on stress-strain relations for isotropic materials	**04**
Q.4	**(a)**	(i). "For bulk modulus to be positive, the value of Poisson's ratio cannot exceed 0.5", Justify.	**04**
		(ii). Define axisymmetric problem with the help of suitable example.	**03**
	(b)	Consider thick walled cylinder subjected to internal pressure and derive expression for radial and circumferential stress for plane stress case.	**07**
		OR	
Q.4	**(a)**	Discuss second theorem of Castigliano or Menabrea's theorem.	**07**
	(b)	Enlist various theories of failure. Discuss maximum elastic strain theory and Octahedral shearing stress theory.	**07**

Q.5 **(a)** Discuss thermo elastic stress-strain relations, equation of equilibrium and strain-displacement relations if thermal stresses are considered. **07**

(b) Following displacement field is imposed on a body; **07**

$U= (xy\boldsymbol{i}+3x^2z\boldsymbol{j}+4\boldsymbol{k})\ 10^{-2}$

Consider point P and a neighboring point Q. Point Q has the direction cosines $n_x = 0.2$; $n_y = 0.8$ and $n_z = 0.555$. Coordinates of point P are (2, 1, 3). If PQ = Δs, find components of P'Q' after deformation.

OR

Q.5 **(a)** Determine radial displacement u_r for solid sphere subjected to purely radial temperature variation. **07**

(b) At point P, the rectangular stress components are; **07**

$\sigma_x = 1$ kPa, $\sigma_y = -2$ kPa, $\sigma_z = 4$ kPa, $\tau_{xy} = 2$ kPa, $\tau_{yz} = -3$ kPa and $\tau_{zx} = 1$kPa find principal stress and check for invariants.

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