*Seat No.: _____* *Enrolment No._____*

# GUJARAT TECHNOLOGICAL UNIVERSITY

**M. E. - SEMESTER – I • EXAMINATION – WINTER • 2014**

**Subject code: 710904N** **Date: 04-12-2014**

**Subject Name: Optimization Techniques**

**Time: 10:30 am - 01:00 pm** **Total Marks: 70**

## Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full mark.**

**Q.1** **(a)** A company has two manufacturing plants located at different places. Each plant produces three different items A, B and C. The production capacities of two plants in number of items per day are as follows: **05**

| | Product A | Product B | Product C |
|---|---|---|---|
| Plant 1 | 3000 | 1000 | 2000 |
| Plant 2 | 1000 | 1000 | 6000 |

A market survey indicates that during any particular month there will be a demand of 24,000 items of type A, 16,000 of type B and 48,000 of type C. The cost per day of running plant 1 and 2 are Rs. 6,000 and Rs 4,000 respectively. How many days should the company run each plant during the month so that the production cost is minimized while still meeting the market demand. Formulate the problem and use graphical method to find the optimal solution.

**(b)** Illustrate graphically: **05**

(a) Infeasible solution (b) Unbounded solution (iii) multiple optimal solutions

**(c)** Write the dual problem of the following primal problem. **04**

Minimize $Z = 2x_1 + 3x_2 + 4x_3$ subject to

$2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2 \geq 0, x_3$ is unrestricted.

**Q.2** **(a)** Solve the following problem using Simplex method. **07**

Maximize $Z = 10x_1 + 15x_2 + 20x_3$ subject to

$10x_1 + 5x_2 + 2x_3 \leq 270$

$5x_1 + 10x_2 + 4x_3 \leq 220$

$x_1 + x_2 + 2x_3 \leq 50$

$x_1, x_2, x_3 \geq 0$

**(b)** A company has three plants at locations X, Y and Z which supply to warehouses located at A, B, C, D, and E. Unit transportation costs (in Rupees), plant capacity and warehouse demands are given below. **07**

| | A | B | C | D | E | Capacity |
|---|---|---|---|---|---|---|
| X | 5 | 8 | 6 | 6 | 3 | 800 |
| Y | 4 | 7 | 7 | 6 | 5 | 500 |
| Z | 8 | 4 | 6 | 6 | 4 | 900 |
| Demand | 400 | 400 | 500 | 400 | 600 | |

Determine an optimum distribution for the company in order to minimize the total transportation cost. Obtain the Basic Feasible Solution (BFS) using VAM and check the BFS for optimality using MODI method.

**OR**

**(b)** A company manufactures LED TVs at two factories X and Y and with production capacities of 200 and 300 units, respectively. The demand at retail stores A, B and C are 100, 150 and 250 units, respectively. Rather than shipping the TVs directly from factories to retail stores, the company is investigating the possibility of transshipment. The transportation cost (in Rs. per unit) is given in the table. Find the optimum shipping schedule. **07**

| | | *Factory* | | *Retail Store* | | |
|---|---|---|---|---|---|---|
| | | *X* | *Y* | *A* | *B* | *C* |
| *Factory* | *X* | 0 | 8 | 7 | 8 | 9 |
| | *Y* | 6 | 0 | 5 | 4 | 3 |
| *Retail Store* | *A* | 7 | 2 | 0 | 5 | 1 |
| | *B* | 1 | 5 | 1 | 0 | 4 |
| | *C* | 8 | 9 | 7 | 8 | 0 |

**Q.3** **(a)** There are 5 jobs namely, *A*, *B*, *C*, *D*, and *E*. These are to be assigned to 5 machines *P*, *Q*, *R*, *S* and *T* to minimize the cost of production. The cost matrix is given below. Assign the jobs to machine on one to one basis. **05**

| | Jobs (cost in Rs.) | | | | |
|---|---|---|---|---|---|
| Machines | *A* | *B* | *C* | *D* | *E* |
| *P* | 8 | 7 | 4 | 11 | 6 |
| *Q* | 10 | 5 | 5 | 13 | 7 |
| *R* | 6 | 9 | 8 | 7 | 12 |
| *S* | 6 | 7 | 2 | 3 | 2 |
| *T* | 7 | 8 | 8 | 10 | 5 |

**(b)** Find the dimensions (radius R and height H) of the cone of maximum volume which can be inscribed in a sphere of radius 2. **05**

**(c)** Define the following types of problems: **04**

(i) Linear programming problem (ii) Quadratic programming problem
(iii) Integer programming problem (iv) Dynamic programming

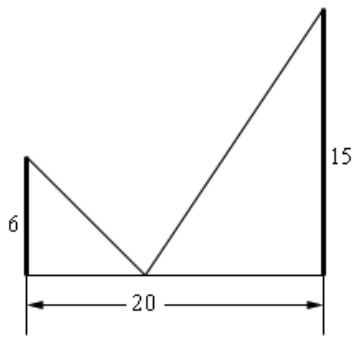
**OR**

**Q.3** **(a)** A salesman wishes to start from a particular city, visit each city only once, and then return to the journey originating point. The travelling distance of each city from a particular city is given in the matrix: **05**

| | To City | | | | |
|---|---|---|---|---|---|
| From City | *A* | *B* | *C* | *D* | *E* |
| *A* | -- | 15 | 22 | 17 | 18 |
| *B* | 15 | -- | 22 | 16 | 17 |
| *C* | 15 | 17 | -- | 19 | 15 |
| *D* | 16 | 19 | 20 | -- | 19 |
| *E* | 19 | 17 | 21 | 20 | -- |

What is the recommended sequence of his visits so that the travelling distance is the minimum?

**(b)** Two poles, one 6 meters tall and other one 15 meters tall, are 20 meters apart. A length of wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. Where shall the wire be staked so that the minimum amount of wire is used? **05**



**(c)** Differentiate between the following with the help of examples and/or neat sketches: **04**

(i) Free point and Bound point

(ii) Linear programming and Non-linear programming

**Q.4** **(a)** Solve the following Geometric Programming problem: **07**

Minimize $f(X) = 16x_1x_2x_3 + \frac{4x_1}{x_2} + \frac{2x_2}{x_3^{\,2}} + \frac{8x_3}{x_1^{\,3}}$

**(b)** Find the optimum integer solution to the following LPP using Cutting Plane method. **07**

Maximize $Z = 5x_1 + 10x_2$ subject to

$-2x_1 + 4x_2 \le 5$

$2x_1 + x_2 \le 11$

$x_1, x_2 \ge 0$ and integers

**OR**

**Q.4** **(a)** Find the maximum of the function $f(X) = -3x_1^{\,2} - 5x_2^{\,2} - 6x_1x_2 + 7x_1 + 5x_2$ subject to $g(X) = x_1 + x_2 = 5$ using the Lagrange multiplier method. **07**

**(b)** Explain the following terminology related to Geometric Programming: **04**

(i) Posynomial (ii) Degree of difficulty.

**(c)** What are different types of integer programming problems? Explain each of them. **03**

**Q.5** **(a)** Find the point P(*x, y, z*) on the plane *2x+y-z=5* that is closest to the origin using (i) Method of substitution, and (ii) Lagrangeĉs multiplier method. **07**

**(b)** Minimize $f(x) = x_1^{\,2} + x_2^{\,2} + 60x_1$ subject to **07**

$x_1 - 80 \ge 0$

$x_1 + x_2 - 120 \ge 0$ using Kuhn-Tucker conditions.

**OR**

**Q.5** **(a)** Use the Golden section method to find the optimum value of $x$ in the interval [0, 2] to minimize the function $f(x) = x^4 - 14x^3 + 60x^2 - 70x$. **07**

Carry out 5 iterations.

**(b)** With reference to dynamic programming, explain the terms: (i) State (ii) Stage (iii) Transformation. What is the *Principle of Optimality*? State the applications of dynamic programming. **07**

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